In 2-dimensions, we must use vectors, so that we write

$$\vec{r}(t+\Delta t) = \vec{r}(t) + \vec{v}(t)\Delta t$$

Why can we still write Δt ?

Acceleration is defined as a vector in the direction of the change of the velocity vector and having a magnitude given by

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} \quad \text{or} \quad a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \quad \text{in 1-dimension}$$

It is a generalization of previous equation for velocity.

Physicists (along with everyone else) like to generalize ideas as long as they can get away with it ... it is easy...and you do not have to think up anything new.

We then have (as with velocity)

$$\vec{v}(t+\Delta t) = \vec{v}(t) + \vec{a}(t)\Delta t$$

Now, suppose I **interact** with 2 different bodies in **"same"** manner, i.e., hang the same object over a pulley and attach it to the 2 bodies with a string.

We define **stuff** in each body by seesaw balancing such that the amount of stuff in 2 bodies is identical if the seesaw balances and ratio of the stuff in two bodies is given by the inverse ratio of the distance from the pivot when the seesaw balances. Stuff = mass = m!

We note that(experiment) says

$$\frac{a_1}{a_2} = \text{constant} = \frac{\text{stuff in 2}}{\text{stuff in 1}} = \frac{m_2}{m_1}$$

Whenever simple results like that come out of an experiment, physicists(Newton and Galileo in this case) say that something profound must be going on here........

In this case, they turned the equations around and said

$m_1 a_1 = m_2 a_2$ --> something to do with my "same" interaction!!

So given the acceleration, we can calculate the velocity and then calculate the position and get the answer we are looking for .... the process uses calculus ...... that is why Newton invented it.

But how do we find the acceleration from first principles .... remember that is what theorists do !

This leads us into a discussion of **Dynamics** ....

Newton's Laws (the crowning achievement of classical physics)

A body at **rest** is not moving! There is no difference between a body at rest and a body moving with constant velocity since we can always change our frame of reference and then the body with constant velocity looks like it is at rest (and the body that was at rest now looks like it has a constant velocity).

A body is **interacting** with its surroundings in some manner when we see a **changing** velocity or an acceleration.

Now push(or pull) on object and watch it accelerate. It is clear that is clear that I can make the body have a smaller or larger acceleration depending on the strength of my interaction with it. It is also clear that my interaction is directional ... it produces a directional or vector quantity ... the acceleration.

This leads to the concept of a **force**. Force is a vector quantity that somehow represents and quantifies my interaction with the body. Since in the earlier experiment, my interaction in the two cases was the "same", I must have been exerting the same force.

This led Newton to postulate the relationship

$$\vec{F} = m\vec{a}$$

so that in the earlier experiment I was exerting the same force!

Be careful here! Is there any new physical content to the introduction of the concept of force or is all the physics contained in the acceleration? I can measure the acceleration! Can I measure the force or do I just infer it from a measured acceleration?

Newton's Laws

(1) an isolated body has no acceleration

(1') a body at rest or moving with constant velocity remains at rest or moving with constant velocity unless it interacts with something

Any real content?

(2) $\vec{F} = m\vec{a}$

Is this simply a definition of the force?

(3) If body A exerts a force on body B , then body B exerts an equal and opposite force on body A (here is real content!)

Energy

Most dynamics problems of the everyday world can be solved using Newton's laws. But they are not suitable for generalization beyond the realm of everyday experience.

In order to find the rules and laws that are appropriate in other regimes of interest like very high speeds (SR) and very small distances (QM) we must find a different way of thinking about the universe. This new way is based on Newton's laws so there is no new physical content, but it will be possible to extend to meaning of the new laws so that new physical content and thus new physical theories can be formulated.

Energy is one of these new concepts that allows generalization.

We first define **kinetic energy** or **energy due to motion** as

$$K = \frac{1}{2}mv^2$$

Now we do an experiment. We take any object raise it up to some height h above the ground and then release it. We find the following relationships:

$$v_{ground}^2 = 2gh \qquad \text{g} = 9.8 \text{ m/s}^2$$
$$v(t) = gt$$
$$y(t) = h - \frac{1}{2}gt^2$$
$$v^2(t) + 2gy(t) = constant$$

This last result is the key. As we said earlier, much of theoretical physics is a search for invariants. We saw a couple in SR. When studying dynamics, invariants are quantities that are constant in time. The last experimental relation can be written

$$\frac{1}{2}mv^2(t) + mgy(t) = constant$$
$$K + V = E$$

where we have defined two new energies

$V = mgy$ = potential energy
$E = K + V$ = total energy

The last experimental result then allows us to postulate

The total energy is a constant of the motion

The kinetic energy K and the potential energy V are not constant during the motion. In fact, they are constantly changing into one another,i.e., there is an exchange between K and V during the motion.

The law that energy is a constant is an example of a conservation law for some invariant quantity. We derive conservation laws from simple experiments and then generalize their validity to a much wider range of phenomena.

Momentum is another one of the new concepts. It is a vector quantity.

It turns out that velocity is not the important dynamic variable. How can we see this?

Suppose we have a hill with two dump trucks at the top. One of the dump trucks is filled with sand and the other is empty.

We know from experiment that the velocity is the same for different trucks when they reach the bottom of the hill or they have the same acceleration ... the acceleration seems to be independent of the amount of stuff in the trucks (a property of the gravitational interaction).

Now we ask this question?

Which of these two trucks would you want to attempt to stop at the bottom of the hill?

Clearly the answer is the truck with the least stuff or the smaller mass.

So we define a new dynamical quantity

$$\vec{p} = m\vec{v} = \text{linear momentum}$$

Newton's second law then becomes

$$\vec{F} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta (m\vec{v})}{\Delta t}$$

which for a constant mass system becomes

$$\vec{F} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t} = m \lim_{\Delta t \to 0} \frac{\Delta (\vec{v})}{\Delta t} = m\vec{a}$$

as before.

Therefore we now restate Newton's laws as:

(1) The linear momentum is conserved for an isolated body $\Delta \vec{p} = 0$

(2) $\vec{F} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t}$

(3) The total momentum of an isolated system is a constant

$$\vec{p}_1 + \vec{p}_2 = constant$$
$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$$
$$\frac{\Delta\vec{p}_1}{\Delta t} = -\frac{\Delta\vec{p}_2}{\Delta t}$$
$$\vec{F}_{12} = -\vec{F}_{21}$$

So for an isolated system we have a **conservation law** for linear momentum, which we can generalize to a much wider range of phenomena.

So, here is the way scientists of that day thought....

The classical universe followed well-defined laws. Everything was, and is, predictable. If we only find the force, know the masses,positions,and velocities of all the objects under consideration at one single time, then all is predictable from then on!!

The universe is a gigantic Newtonian clockwork. Cause and effect rule. Nothing is by chance. Everything is ultimately accountable.

Perfect determinism. The laws of physics are to be obeyed, because it is impossible to disobey them. The is no room for free will, salvation and damnation, or love and hate. Even the most trifling thought has been determined long ago. You might have imagined that you are a free-thinking person, but even that imagination is nothing but the universal clockwork turning in some yet-to-be-discovered way. So now you are probably thinking...glad they found out those ideas were wrong and got rid of them! Just remember it is always dangerous to make quick judgements like that, especially when you are not sure what will come along to replace it.

And then there was light.......... and Special Relativity

Our derivation of SR has shown that:

(1) We lose position and time as separate quantities, which is an indication that everyday experience may not carry over into these new realms.

Why didn’t physicists notice before? It generally is simply a matter of the accuracy and precision available to experimentalists, i.e., prior to this century, experimental measurements of the speed of light could not say that it was not infinite. If it were infinite, then SR would reduce to GR and Newtonian physics would still be valid.

(3) We must choose our observables with some care.....

(4) We must use conservation laws to give us the physical quantities that represent really what we can know about systems.

(5) We can fully extend classical physics validity to all speeds.

(6) We must rethink our world view (happens all the time in physics)

Everyday experience cannot be our guide

Fine for world of everyday objects

We must be prepared to give up preconceived ideas because they are based on our experiences

We must trust measurements to tell us what is going on but we must define them carefully

But classical physics still hangs on, albeit modified.......

Everything works so well ....

What does that statement really means to a physicist..........

We only know what we measure .... philosophy?

In this world motion is a continuous blend of changing positions. The object moves in a flow from one point to another.

Science is a reasonable, orderly process of observing nature and describing the observed “objectively”.

There is a conviction that whatever one observed as being out there was really out there. The idea of objectivity being absent from science is abhorrent to any rational physicist.

One firmly believes in the passive(non-disruptive) observer. Humans are creatures of the eye. They believe what they see.

So summarizing, classically

[1] Things moved in a continuous manner.
[2] Things move for reasons. The reasons are earlier causes and all motion was determined and predictable.
[3] All motion could be analyzed or broken down into its component parts. Each part played a role in the giant machine called the universe. The complexity of this machine could be understood in terms of the simple movement of its various component parts.
[4] The observer observed and never disturbed. All experimental

errors could be analyzed and understood.

All of these ideas will turn out to be false!!!!!

Now back to SR.

What happens to momentum and energy when we enter the realm of SR?

At this level we must rely on experiments to point the proper way to proceed. In Physics 7/8 some of you will derive these results from first principles using the interval and linear algebra.

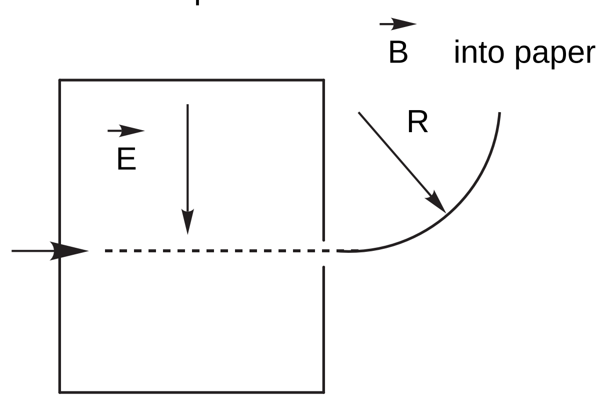
The following result has been confirmed by experiment.

The force felt by a charged particle in electric and magnetic fields is given by the Lorentz force law

$$\vec{F} = q\left(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}\right)$$

where $\vec{v}$ is the particle velocity.

Consider the experimental setup below.



In the box region the electric and magnetic fields are adjusted so that $\vec{F} = 0$ for a particle moving along the dotted line with a definite velocity. The electric force always points downward and the magnetic force is always perpendicular to the velocity direction(upward in the box for a particle moving along the dotted line). This means that particles with a particular velocity, namely,

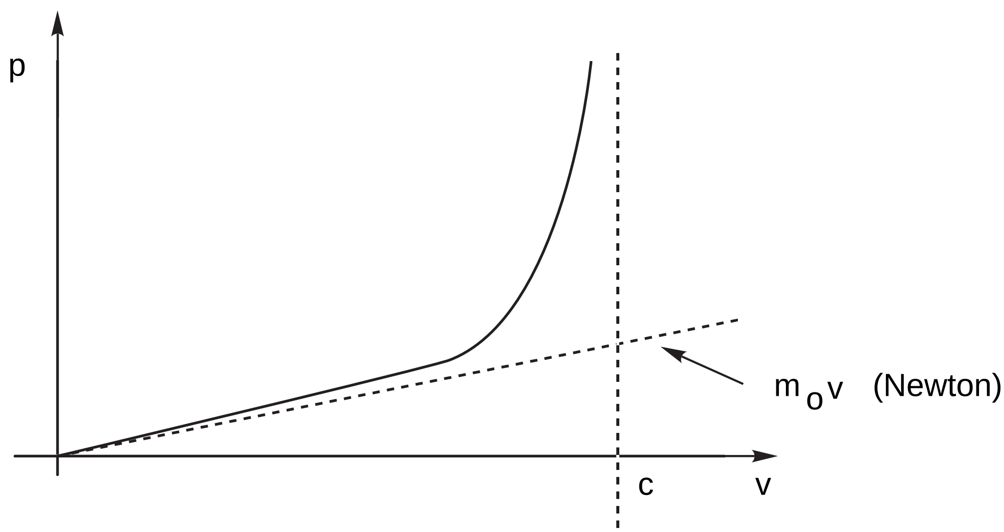
$$q(-E + \frac{v}{c}B) = 0 \rightarrow \frac{v}{c} = \frac{E}{B}$$

pass undeflected though the box. The box is called a velocity selector. Outside the box there is no electric field, so the particle moves on a circular path (Force always perpendicular to the velocity). In plane polar coordinates with a radius of

$$R = \frac{pc}{qB}$$

So that measuring the radius corresponds to measuring the relativistic momentum. Thus, in the same experiment we can measure **both** the velocity and momentum **independently** and thus determine the relationship between them.

A plot of the experimental results looks like



This corresponds to the result

$$p = \gamma m_0 v \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

instead of the Newtonian assumption that

$$p = m_0 v$$

where m_0 = the so-called rest mass. It is the only valid mass for a particle since we measure mass when a body is at rest. Any measurement of mass when a particle is moving is really a measurement of its momentum and thus it would be incorrect for us to assume that any different mass value can be used for a moving object. Thus, there is no such thing as the **"relativistic mass"**.

Now what about relativistic energy? What is the relativistically correct form of the energy of a particle?

One way to generalize the concept of energy is to use the Newtonian definition of kinetic energy in conjunction with the relativistically correct definition of momentum. The derivation that follows uses calculus. Do not worry if you cannot follow all of the steps (you will be able to follow them when the derivation is repeated in

Physics 7). For this course only the results are important. We proceed as follows:

The formal definition of kinetic energy is given as

$$\Delta K = K - K_0 = \text{work done by force} = \int_{\vec{r}_0}^{\vec{r}} \vec{F}\cdot d\vec{r} = \int_{\vec{r}_0}^{\vec{r}} \frac{d\vec{p}}{dt}\cdot d\vec{r}$$

We found that $\vec{p} = m_0\gamma(v)\vec{v}$ where $\gamma(v) = \left(1-\beta^2\right)^{-1/2}$, $\beta = \frac{\mathrm{v}}{\mathrm{c}}$. Therefore we have

$$K - K_0 = \int_{\vec{r}_0}^{\vec{r}} \frac{d}{dt}(m_0\gamma(v)\vec{v})\cdot\vec{v}dt = m_0\int_0^v \vec{v}\cdot d(\gamma(v)\vec{v})$$

Since the kinetic energy is zero when the velocity is zero we finally have

$$K = m_0\int_0^v \vec{v}\cdot d(\gamma(v)\vec{v})$$

Now since

$$d(\gamma v^2) = d(\gamma\vec{v}\cdot\vec{v}) = \vec{v}\cdot d(\gamma\vec{v}) + \gamma\vec{v}\cdot d\vec{v}$$

we can write

$$K = m_0\int_0^v (d(\gamma v^2) - \gamma\vec{v}\cdot d\vec{v}) = m_0\int_0^v d(\gamma v^2) - \frac{1}{2}m_0\int_0^v \gamma d(v^2)$$

$$= m_0\gamma v^2 - \frac{1}{2}m_0c^2\int_0^{v^2/c^2} \gamma du\frac{du}{\sqrt{1-u}} = m_0\gamma v^2 + m_0c^2\left(\frac{1}{\gamma}-1\right)$$

$$= m_0c^2\left(\gamma\beta^2 + \frac{1}{\gamma}\right) - m_0c^2 = m_0c^2(\gamma - 1)$$

The first thing we should do is check that this makes sense. What is the low velocity limit of this expression?

Using

$$\gamma = \left(1-\beta^2\right)^{-1/2} \to 1 + \frac{1}{2}\beta^2 = 1 + \frac{1}{2}\frac{v^2}{c^2}$$

we have

$$K = m_0c^2(\gamma - 1) \to m_0c^2\frac{1}{2}\frac{v^2}{c^2} = \frac{1}{2}m_0v^2$$

as expected.

If we rearrange this result we have

$$\gamma m_0c^2 = K + m_0c^2 = Energy(motion) + Energy(rest) = Total\ Energy = E$$

It is only the total energy that is conserved!

We thus obtain Einstein’s famous relation $E_{rest} = m_0 c^2$.

What is the connection to momentum? Some algebra gives the following results for relativistic objects:

$$\frac{pc}{E} = \frac{\gamma m_0 v c}{\gamma m_0 c^2} = \frac{v}{c} = \beta \qquad \text{and} \qquad \left(\frac{E}{c}\right)^2 - \vec{p}^2 = m_0^2 c^2 = \text{invariant}$$

Some questions arise:

How come we do not notice the rest energy in everyday experience?

Some numbers:

$$\text{typical kinetic energy} = 0.5(1)(1)^2 \approx 1 \text{ Joule}$$

$$\text{typical rest energy} = (1)(3\text{x}10^8)^2 \approx 10^{17} \text{ Joules}$$

We typically ignore the significantly larger quantity!! The reason for this is that in everyday situations the rest energy does not change; all the same mass is remains in the system at all times. Thus, the rest energy is not a source of possible energy to do other things.

However, in microscopic systems like atoms and nuclei, etc, the rest mass changes in many interactions and thus this energy becomes available for other purposes. Two examples are nuclear fission and fusion.

Are there any new predictions we can make from these results?

The two relations above make the following interesting prediction:

$$v = c \rightarrow \beta = 1 \rightarrow E = pc$$

$$\left(\frac{E}{c}\right)^2 - \vec{p}^2 = 0 = m_0^2 c^2$$

or the only objects that can travel at the speed of light must have a rest mass equal to zero! However, even though they have a zero rest mass, they still possess energy and momentum defying the classical equations!

Such a particle has been observed .... it is the photon or the particle of light.

The derivation that follows uses calculus. Do not worry if you cannot follow all of the steps (you will be able to follow them when the derivation is repeated in Physics 7). For this course only the results are important. We proceed as follows:

In general, when the velocity is changing both its magnitude and direction we have

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m_0\gamma(v)\vec{v})}{dt} = m_0\frac{d}{dt}\left(\frac{\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

Rectilinear Motion

$$F = m_0\frac{d}{dt}\left(\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}\right) = m_0\frac{dv}{dt}\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right) + m_0 v\frac{-\frac{1}{2}(-2\frac{v}{c^2})\frac{dv}{dt}}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

$$= m_0\frac{dv}{dt}\frac{1}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} = m_0\gamma^3\frac{dv}{dt}$$

Newton's law is modified by the factor γ^3 which has a dramatic effect as $v \to c$. Now suppose that we have a constant force F = constant. We can then integrate the equation as follows:

$$Fdt = m_0\gamma^3(v)dv \to Ft = m_0\int_0^v \gamma^3(v)dv$$

Now

$$\frac{d}{dv}(\gamma v) = \gamma + v\frac{d\gamma}{dv}$$

$$\frac{d\gamma}{dv} = \frac{d}{dv}\left(1-\frac{v^2}{c^2}\right)^{-1/2} = \frac{\frac{v}{c^2}}{\left(1-\frac{v^2}{c^2}\right)^{31/2}} = \gamma^3\frac{v}{c^2}$$

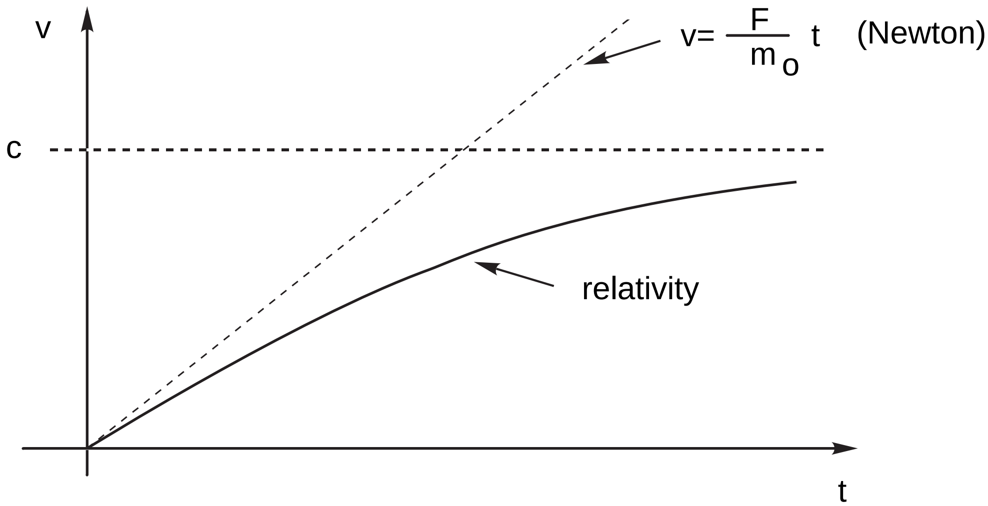
$$\frac{d}{dv}(\gamma v) = \gamma + \gamma^3\frac{v^2}{c^2} = \gamma^3\left(\frac{1}{\gamma^2} + \frac{v^2}{c^2}\right) = \gamma^3$$

Therefore,

$$Ft = m_0\int_0^v d(\gamma v) = m_0\gamma v = m_0\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$F^2t^2 = m_0^2\frac{v^2}{1-\frac{v^2}{c^2}} \to v^2 = \frac{\left(\frac{Ft}{m_0}\right)^2}{1+\left(\frac{Ft}{m_0c}\right)^2} \to v = \frac{dx}{dt} = \frac{F}{m_0c}\frac{ct}{\sqrt{1+\left(\frac{F}{m_0c}\right)^2t^2}}$$

A plot of v versus t is shown below.



It is clear that no matter how long a constant force is applied we still have $v < c$. Continuing the integration

$$dx = \frac{F}{m_0 c}\frac{ct}{\sqrt{1+\left(\frac{F}{m_0 c}\right)^2 t^2}}dt \rightarrow x = \frac{F}{m_0}\int_0^t \frac{t}{\sqrt{1+\left(\frac{F}{m_0 c}\right)^2 t^2}}dt = \frac{F}{m_0}\left(\frac{m_0 c}{F}\right)^2 \times \int_0^{Ft/m_0 c} \frac{u}{\sqrt{1+u^2}}du$$

$$x = \frac{m_0 c^2}{F} \times \int_0^{Ft/m_0 c} d(\sqrt{1+u^2}) = \frac{m_0 c^2}{F}\left(\sqrt{1+\left(\frac{F}{m_0 c}\right)^2 t^2} - 1\right)$$

A plot of x versus t is shown below: