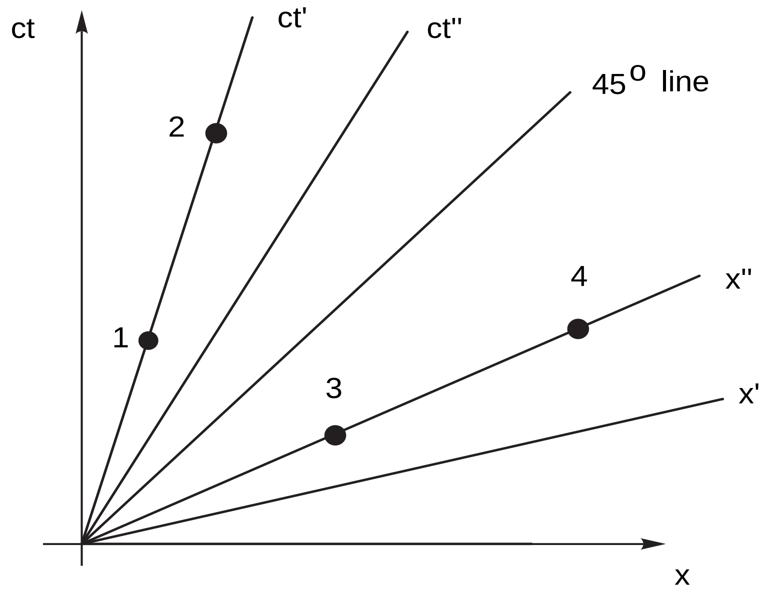
Why are intervals called timelike or spacelike?

From the diagram below it is clear that:



(1) For any timelike pair of events (1 and 2) it is possible to find some observer (coresponding to a new ct'-axis) such that the two events takes place at the same location and hence represent a pure time interval. Hence the name **timelike**.

2) For any spacelike pair of events (3 and 4) it is possible to find some observer (a new x'-axis) such that the two events takes place simultaneously and hence represent a pure space interval. Hence the name **spacelike**.

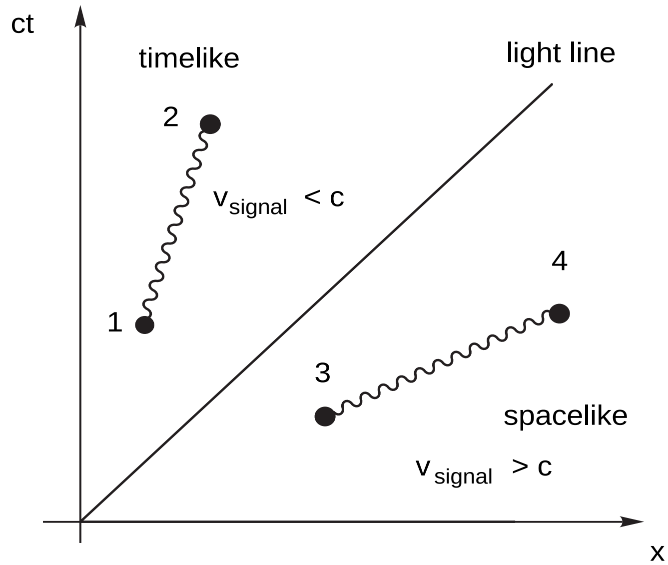
Timelike and spacelike events are radically different. As the diagram below clearly shows:

Event #2, which is timelike relative to event #1, is in the future or forward light cone of event #1

Event #4, which is spacelike relative to event #3, is in the elsewhere region of event #3

Events 1 and 2 can be connected with a signal traveling with a speed less than that of light.

Events 3 and 4 require a signal speed greater than that of light.



Let us explicitly show the invariance of the spacetime interval:

Suppose that we have two events with unprimed coordinates

$$x_1 = 2.0\,,\, ct_1 = 1.0 \quad ; \quad x_2 = 4.0\,,\, ct_2 = 2.0$$
$$\Delta x = x_2 - x_1 = 2.0 \quad ; \quad c\Delta t = c(t_2 - t_1) = 1.0$$

and we assume that $\beta = 0.8 \rightarrow \gamma = \dfrac{1}{\sqrt{1-\beta^2}} = 1.67$. Using the Lorentz transformations we have

$$x'_1 = \gamma(x_1 - \beta ct_1) = 1.67(2.0 - 0.8(1.0)) = 2.00 \;,\; ct'_1 = \gamma(ct_1 - \beta x_1) = 1.67(1.0 - 0.8(2.0)) = -1.00$$
$$x'_2 = \gamma(x_2 - \beta ct_2) = 1.67(4.0 - 0.8(2.0)) = 4.00 \;,\; ct'_2 = \gamma(ct_2 - \beta x_2) = 1.67(2.0 - 0.8(4.0)) = -2.00$$
$$\Delta x' = \gamma(\Delta x - \beta c\Delta t) = 1.67(2.0 - 0.8(1.0)) = 2.00 \;,\; c\Delta t' = \gamma(c\Delta t - \beta\Delta x) = 1.67(1.0 - 0.8(2.0)) = -1.00$$

Therefore,

$$(\Delta S)^2 = (c\Delta t)^2 - (\Delta x)^2 = 1.00 - 4.00 = -3.00$$
$$(\Delta S')^2 = (c\Delta t')^2 - (\Delta x')^2 = 1.00 - 4.00 = -3.00$$

The interval has the same numerical value, even though the time order between the two events is reversed!!!!

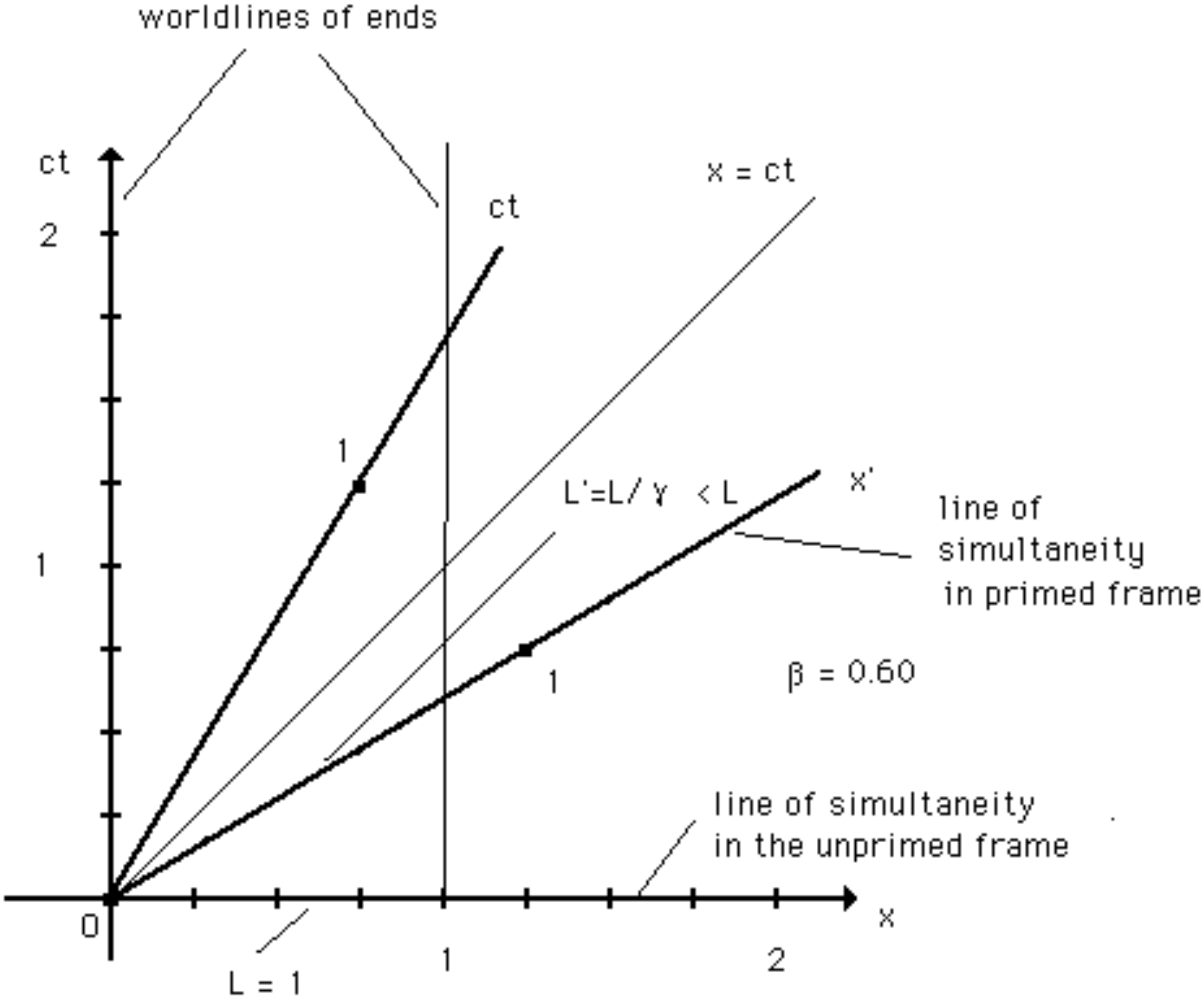
Measurements in Special Relativity

Now let us turn to the measurement properties of spacetime, in particular, the measurement of length and time. First, we need to restate the definitions we decided on earlier:

Length of an object = spatial separation of the two events representing the endpoints of an object measured **simultaneously**(the two events are on a line of simultaneity in a given frame)

Time interval between two events = time separation of the two events measured by a clock **at rest** with respect to the two events (the two events are on the worldline of the clock)

With these definitions we can represent these measurements as follows. Suppose we have two events (ct_1, x_1) and (ct_1, x_2) that correspond to the events of the worldlines of the endpoints of the object being measured, crossing a line of simultaneity (see diagram below). Then the length of the object is given by $L = x_2 - x_1$.

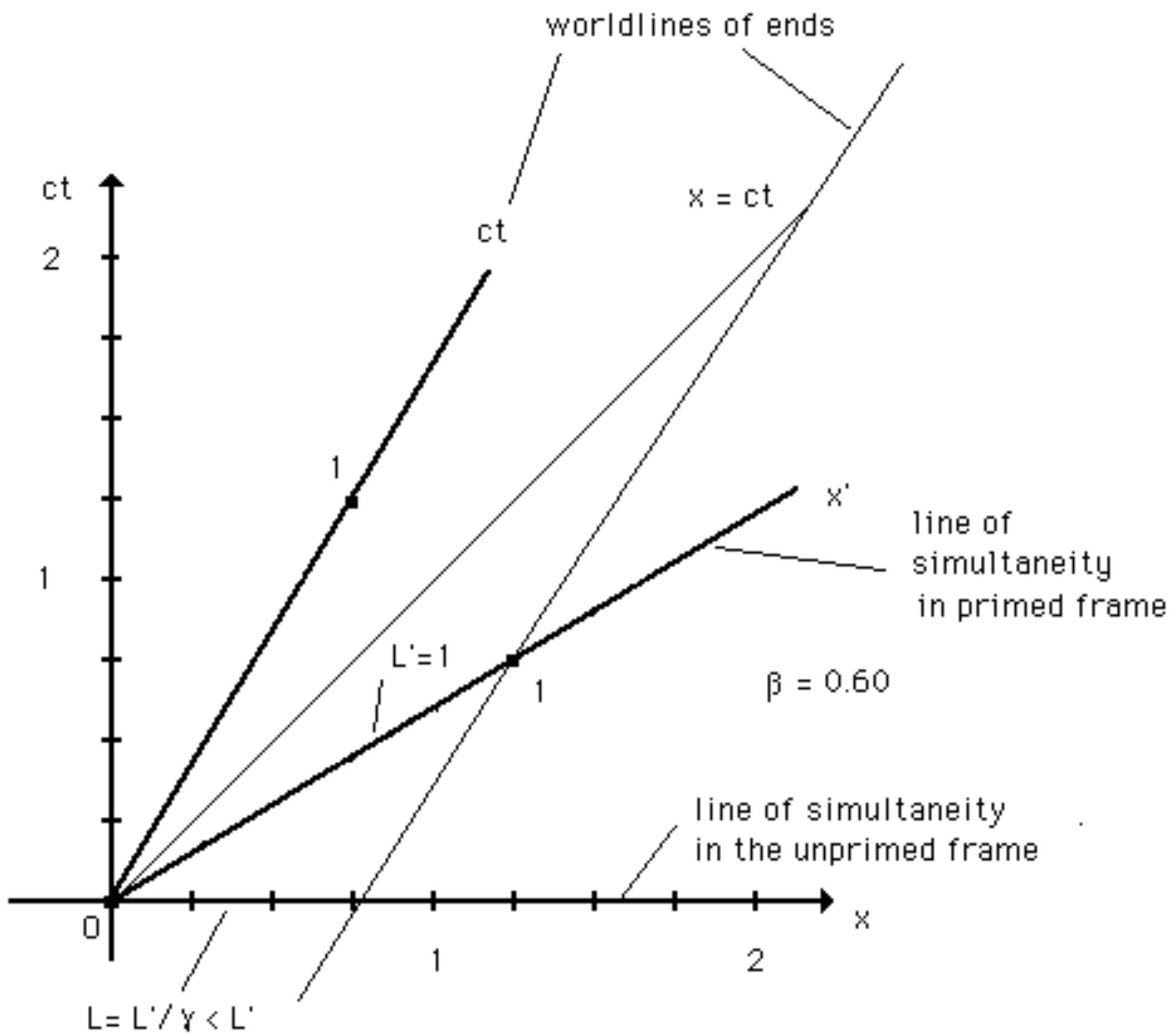


We note as shown in the diagram below that this is not the length as measured in the other reference frame. In fact, $L' = x'_2 - x'_1 = L/\gamma < L$, which is the famous "**length contraction**". Do not be deceived by it looking longer, remember the scales are different.

The **proper length** is the length measured in the object's rest frame (the unprimed frame in this case, because that is where the endpoint worldlines are **parallel** to the time axis, which is the definition of being at rest). The **proper length** is the **maximum measured length**.

We note that we have not said that any object has physically **"contracted"**, but instead we have said its measured length is less! The measured length is less because the two observers **do not agree about simultaneity**, i.e., they have different lines of simultaneity. So even though we use the word **"contraction"**, we must understand that the effect is due to a disagreement about simultaneity and **no physical contraction has actually occurred**.

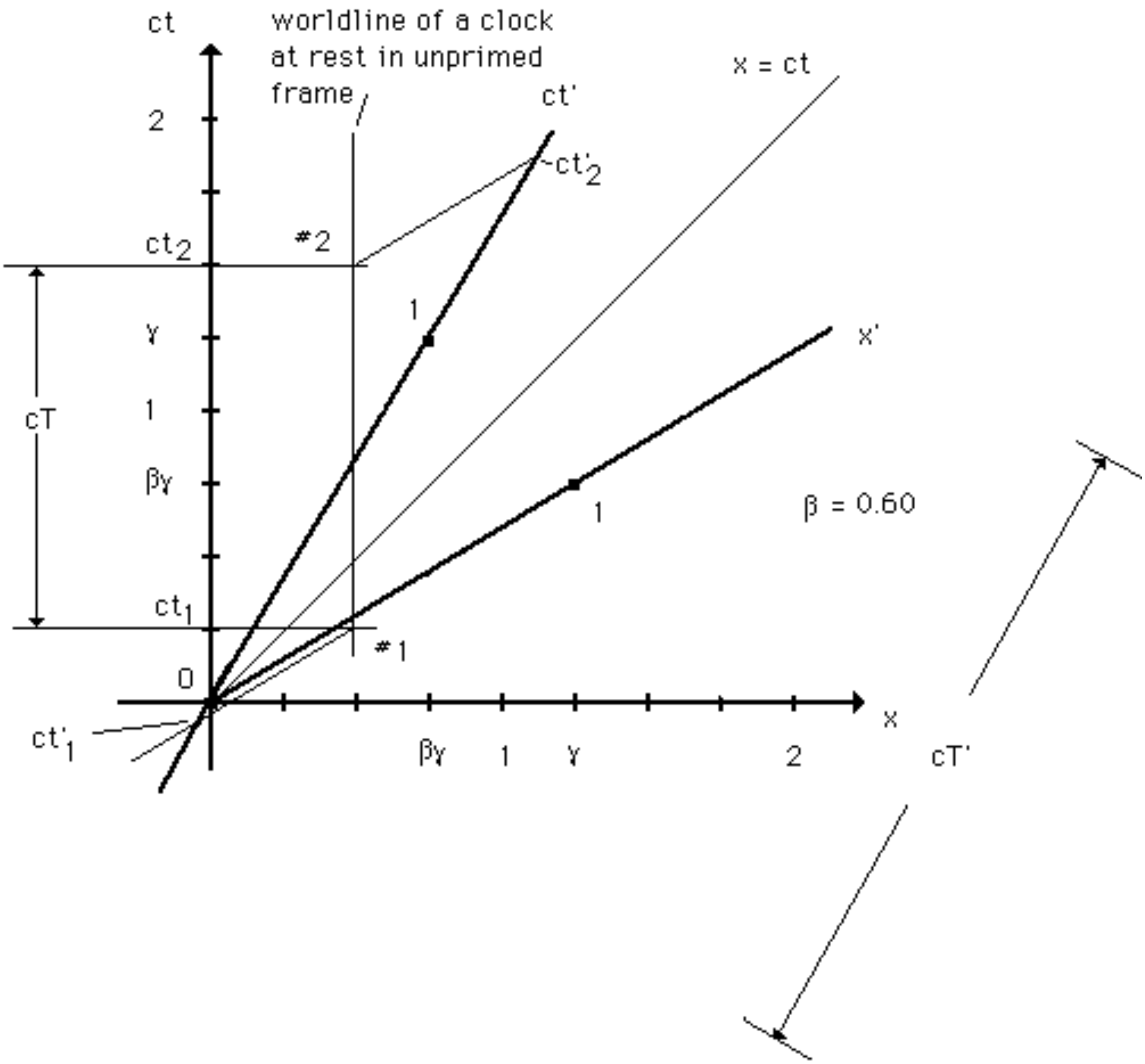
If the object is at rest in the primed frame, then we get an identical result just exchanging the roles of the two frames. As can be seen from the diagram below, in this case, $L = x_2 - x_1 = L'/\gamma < L'$.



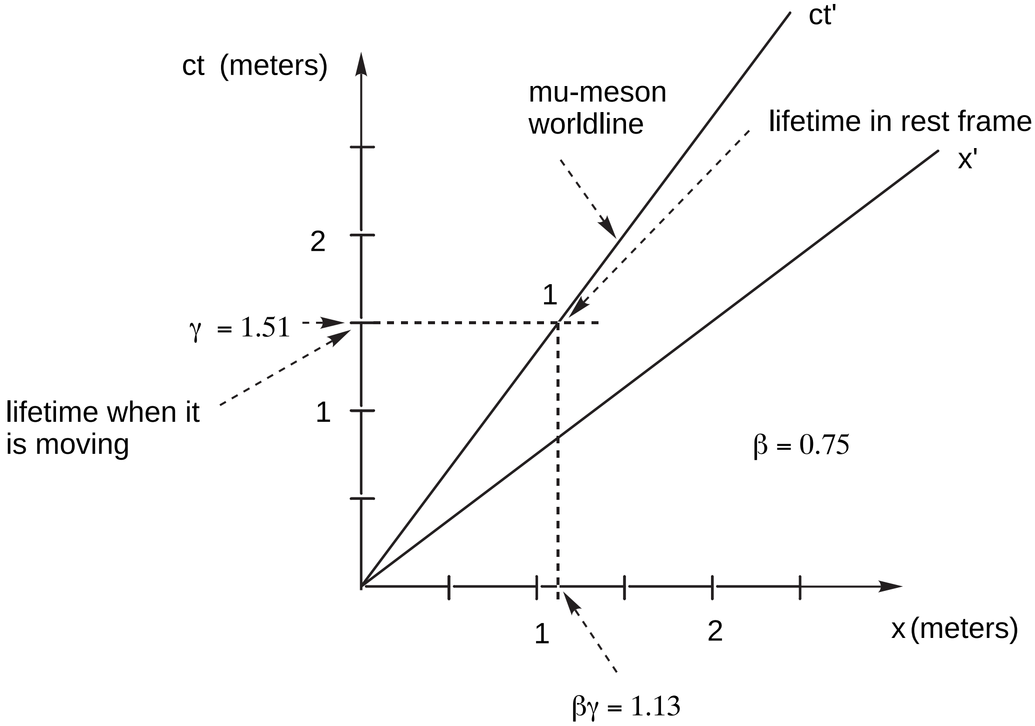
Time dilation is handled in the same way.

Consider the diagram below representing a system that is at rest in the unprimed frame and only lives for a finite amount of time(like mu-mesons). The **proper time** interval for this system is the time separation T between the events (its birth(event #1) and its death(event #2)) as measured by a clock at rest with respect to the system or, in this case, at rest in the unprimed frame.

As can be seen from the diagram the time separation for an observer in the primed frame is $T' = \gamma T > T$. The **proper time** is the **shortest time interval**.



This result is identical to the mu-meson experiment we discussed earlier, which was just an example of time-dilation as can be seen from the diagram reproduced below:



where in this case,

$$T' = 1 \quad \text{and} \quad T = \gamma T' = \gamma$$

We can also see both of these results directly using the Lorentz transformations or the invariance of the interval.

Lorentz Transformations

Length Contraction

The relevant events representing on the worldlines of the ends of an object are

$$(x_1, ct_1) = (0.0, 0.0) \text{ and } (x_2, ct_2) = (1.0, 0.0) \quad \text{for unprimed observer}$$

$$(x_1, ct_1) = (0, 0) \text{ and } (x_3, ct_3) = (1.0, \beta) = (1.0, 0.60) \quad \text{for primed observer}$$

Where are these events on a diagram?

Then the length this object, by definition, is the spatial separation along a line of simultaneity for the unprimed observer

$$L = x_2 - x_1 = \Delta x = 1$$