Physics 6H   Freshman Seminar   Fall 2004   Assignment #6

Readings:

- Boccio - Classical Breakdown
- Boccio - Quantum Thoughts
- Boccio - Mathematics Sections I and II (pp 1-16)
    - Probability
    - Complex Numbers
    - Series
    - Simple Differential Equations(ODEs)
- Boccio - Calculus Notes (if you need a review)
- Boccio - ODE Notes

Summary: This week we will be reading about and discussing

- The Breakdown of Classical Physics
- First Thoughts about the Quantum World
- Mathematics - Probability, Complex Numbers, Series and ODEs

We will do lots of problems since this is the only way to learn physics.

Everyone Problems:

EP-2.	More Balls in a Box
EP-8(a,b,c,d,e).	Complex Numbers
EP-9(a,b,c,d).	Complex Exponentials
EP-10(a,b,c,d).	Complex Expressions
EP-10a(a,b).	Series
EP-10b(a,b).	ODEs

Individual Problems:

EP-1.	Balls in a Box
EP-3.	Playing Cards
EP-4.	Birthdays
EP-5.	Bayes
EP-6.	Is There Life?
EP-7.	Law of Large Numbers
EP-8(f).	Complex Numbers
EP-9(e).	Complex Exponentials
EP-10(e).	Complex Expressions
EP-10(f).	Complex Expressions
EP-10a(c,d).	Series
EP-10b(c).	ODEs

Presentations: Questions and discussion

- Blackbody radiation
- Photoelectric effect
- Interference and diffraction
- Wave-Particle duality
- Probability

Seminar Break:

Extra Problems:

EP-1. Balls in a Box - Suppose the box contains $N = 50$ balls, each bearing an integer between 1 and 8; Letting n_k be the number of balls showing the value $\nu_k = k$, suppose that

$$n_1 = 3\,, n_2 = 2\,, n_3 = 5\,, n_4 = 8\,, n_5 = 13\,, n_6 = 9\,, n_7 = 6\,, n_8 = 4$$

use the probability concepts we have developed to calculate the probability that the numbers found on two random samplings will sum to 6. [answer = $135/2500$]

EP-2. More Balls in a Box - Consider the collection of numbered balls described in EP-1.

(a) Calculate $\langle \nu \rangle$ and $\Delta\nu$ [answer: $\langle \nu \rangle$= 4.94 and $\Delta\nu$=1.8]

(b) Sketch a "frequency bar-graph" of the expected results of $M = 100$ samplings [i.e., lay out the values ν_k on the horizontal axis, and construct vertical "bars" to indicate the number of times each ν_k-value should be obtained]. Show on the graph by means of a vertical line the value $\langle \nu \rangle$. Also, draw a horizontal line of length $2\Delta\nu$ in such a way that it indicates roughly the "spread" or "dispersion" of ν-values about $\langle \nu \rangle$.

EP-3. Playing Cards - Two cards are drawn at random from a shuffled deck and laid aside without being examined. Then a third card is drawn. Show that the probability that the third card is a spade is 1/4 just as it was for the first card.

HINT: Consider all the (mutually exclusive) possibilities (two discarded cards spades, third card spade or not spade, etc).

EP-4. Birthdays - What is the probability that you and a friend have different birthdays? (for simplicity let a year have 365 days). What is the probability that three people have different birthdays? Show that the probability that n people have n different birthdays is

$$p = \left(1 - \frac{1}{365}\right)\left(1 - \frac{2}{365}\right)\left(1 - \frac{3}{365}\right).....\left(1 - \frac{n-1}{365}\right)$$

Estimate this for $n << 365$ by calculating $\ell n(p)$ (use the fact that $\ell n(1+x) \approx x$ for $x << 1$). Find the smallest integer n for which $p < 1/2$. Hence show that for a group of 23 people or more, the probability is greater than 1/2 that two of them have the same birthday.

EP-5. Bayes - Suppose that you have 3 nickels and 4 dimes in your right pocket and 2 nickels and a quarter in your left pocket. You pick a pocket at random and from it select a coin at random. If it is a nickel, what is the probability that it came from your right pocket? Use Baye's formula.

EP-6. Is There Life?

The number of stars in our galaxy is about $N = 10^{11}$. Assume that the probability that a star has planets is $p = 10^{-2}$, the probability that the conditions on the planet are suitable for life is $q = 10^{-2}$, and the probability of life evolving, given suitable conditions, is $r = 10^{-2}$. These numbers are rather arbitrary.

(a) What is the probability of life existing in an arbitrary solar system (a star and planets, if any)?

(b) What is the probability that life exists in at least one solar system?

NOTE: A naive argument against a purely natural origin of life is sometimes based on the smallness of the probability (a), whereas it is the probability (b) that is relevant!

EP-7. Law of Large Numbers

This problem illustrates the law of large numbers (use IDL).

(a) Assuming the probability of obtaining "heads" in a coin toss is 0.5, compare the probability of obtaining "heads" in 5 out of 10 tosses with the probability of obtaining "heads" in 50 out of 100 tosses.

(b) For a set of 10 tosses and for a set of 100 tosses, calculate the probability that the fraction of "heads" will be between 0.445 and 0.555.

EP-8. Complex Numbers - Prove the following properties of complex numbers:

(a) $\mathrm{Re}\, z = \dfrac{z + z^*}{2}$, $\mathrm{Im}\, z = \dfrac{z - z^*}{2i}$

(b) z is a pure real number if and only if $z^* = z$ and
z is a pure imaginary number if and only if $z^* = -z$

(c)

$$z^{**} = z$$
$$(z_1 + z_2)^* = z_1^* + z_2^*$$
$$(z_1 z_2)^* = z_1^* z_2^*$$

(d) $|z|^2 = (\mathrm{Re}\, z)^2 + (\mathrm{Im}\, z)^2$

(e) $|z| \geq |\mathrm{Re}\, z|$ and $|z| \geq |\mathrm{Im}\, z|$

(f) $|z_1 + z_2| \leq |z_1| + |z_2|$

EP-9. Complex Exponentials - Prove the following relations:

(a) $\left(e^{ikx}\right)^* = e^{-ikx}$

(b) $e^{ik_1 x}e^{ik_2 x} = e^{i(k_1+k_2)x}$

(c) $\left|e^{ikx}\right|^2 = 1$

(d) $\frac{d}{dx}e^{ikx} = ike^{ikx}$

(e) $\int e^{ikx}dx = \frac{1}{ik}e^{ikx} + C$

EP-10. Complex Expressions - Evaluate the following expressions with your final answers in the form $z = a + bi$.

(a) $z = (2+3i)^3$

(b) $z = e^{i\pi/3}$

(c) $z = i^i$

(d) $z = \frac{1}{i-1}$

(e) $z = \sin\left(\frac{\pi}{2} + i\ell n(2)\right)$

(f) $z = (1+i)^{1-i}$

EP-10a. Series - Write the first five terms in a series expression for the following:

(a) $\frac{e^x}{1-x}$

(b) $\frac{\sin\sqrt{x}}{\sqrt{x}}$

(c) $\ell n(1+x)$

(d) $\sin(\ell n(1+x))$

EP-10b. ODEs - Find solutions to the following ODEs:

(a) $\frac{dy}{dt} = -12y \quad , \quad y(3) = 1$

(b) $4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 48y = 0 \quad , \quad y(0) = 0\,,\ \frac{dy}{dx}(0) = 7$

(c) $\frac{d^2y}{dx^2} + 16y = 0 \quad , \quad y(0) = 2\,,\ \frac{dy}{dx}(0) = 8$

Rewrite the answer to (c) in terms of sines and cosines.