

Readings:

- Albert - Chapter 4
- Albert - Chapter 5
- Boccio - EPR and Bell Details
- Boccio - More EPR
- Boccio - Whither the Collapse?

This week we delve even deeper into EPR and Bell and then look at the collapse process in depth.

Everyone Problems:

- EP-72 EPR
- EP-77 Ammonia Molecule

Individual Problems:

- EP-73 MoreEPR redux
- EP-75 Angular Momentum
- EP-76 Angular Momentum Measurement
- EP-78 Angular Momentum

Presentations:

- Single Photon Interference
- Inseparable Photons and the EPR Paradox
- The BCHSH Inequality
- Proof of the Bell Inequality
- Experiments at Orsay
- Bell's Theorem with Photons
- MoreEPR
- Collapse

Seminar Break:

Extra Problems:

EP-72. EPR - Let the particles entering the ingenious color box described in the MoreEPR handout be $\begin{pmatrix} -1/2 \\ -\sqrt{3}/2 \end{pmatrix}$.

- Work out the probability that a particle emerges from the + output when the box is in each of its three configurations.
- Work out the probability that a particle emerges from the - output when the box is in each of its three configurations.
- Show that the probability for either output is 1/2 if the box randomly switches between the three configurations.

EP-73. MoreEPR redux - In the "more complete" theory described in the EPR handout, imagine that the one particle of the pair heading toward box #1 carries instructions (-+-).

- Work out from which output of each box particles emerge in all 9 possible configuration of the two boxes.

- (b) For a particle with this instruction set, calculate the probability that particles will emerge from different outputs of the two boxes.

EP-75. Angular Momentum - When measuring the angular momentum along the z-direction of the atoms in problem 74, there are some states for which the average value of the measurements is zero.

- (a) Find the probabilities of measuring each eigenvalue of \hat{O}_{L_z} for the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$.
- (b) Use these probabilities to show that the average measurement is zero.
- (c) Is there a state for which the average value of the measurements of the x-, y-, and z-components of the angular momentum are all zero? Show that the answer is no by choosing a general state $|\psi\rangle = \alpha|1\rangle + \beta|2\rangle$ and forcing the probabilities to be such that the average measurement of all three components is zero, yielding conditions on α and β that can only be satisfied if $\alpha = \beta = 0$.

EP-76. Angular Momentum Measurement - A measurement of a component of the intrinsic angular momentum (or spin) of an electron always yields one of two values, namely, $\pm\hbar/2$. If we call the component we are measuring the z-component, we can easily form an operator representing this measurement (call it \hat{O}_{S_z}) and the eigenstates representing the two states resulting from the measurement.

$$\hat{O}_{S_z} = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad | +z \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad | -z \rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

In the $(| +z \rangle, | -z \rangle)$ basis, the operator representing measurements of spin along the y-axis is

$$\hat{O}_{S_y} = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

- (a) Find the eigenvalues and associated eigenvectors of \hat{O}_{S_y} .
- (b) What state (call it $|X\rangle$) results when the $| +z \rangle$ state is operated on by the operator

$$\hat{O}_X = e^{-i\theta\hat{O}_{S_y}}$$

where θ is a constant?

- (c) What are the possible outcomes of a measurement of the z-component of the spin of an electron in the state $|X\rangle$ and with what probability will each occur?
- (d) What are the possible outcomes of a measurement of the y-component of the spin of an electron in the state $|X\rangle$ and with what probability will each occur?

EP-77. Ammonia Molecule

In the ammonia molecule, NH_3 , the three hydrogen atoms lie in a plane

at the vertices of an equilateral triangle. The single nitrogen atom can lie either above or below the plane containing the hydrogen atoms, but in either case the nitrogen atom is equidistant from each of the hydrogen atoms (they form an equilateral tetrahedron). Let us call the state of the ammonia molecule when the nitrogen atom is above the plane of the hydrogen atoms $|1\rangle$. Let us call the state of the ammonia molecule when the nitrogen atom is below the plane of the hydrogen atoms $|2\rangle$.

How do we determine the energy operator for the ammonia molecule?

If these were the energy eigenstates, they would clearly have the same energy (since we cannot distinguish them in any way). So diagonal elements of the energy operator must be equal if we are using the $(|1\rangle, |2\rangle)$ basis. But there is a small probability that a nitrogen atom above the plane will be found below the plane and vice versa (called tunnelling). So the off-diagonal element of the energy operator must not be zero, which also reflects the fact that the "above" and "below" states are not energy eigenstates. We therefore arrive with the following matrix as representing the most general possible energy operator for the ammonia molecule system:

$$\hat{H} = \begin{pmatrix} E_0 & A \\ A & E_0 \end{pmatrix}$$

where E_0 and A are constants.

- (a) Find the eigenvalues and eigenvectors of the energy operator. Label them as $(|I\rangle, |II\rangle)$.
- (b) Let the initial state of the ammonia molecule be $|I\rangle$, that is $|\psi(0)\rangle = |I\rangle$. What is $|\psi(t)\rangle$, the state of the ammonia molecule after some time t ? What is the probability of finding the ammonia molecule in each of its energy eigenstates? What is the probability of finding the nitrogen atom above or below the plane of the hydrogen atoms?
- (c) Let the initial state of the ammonia molecule be $|1\rangle$, that is $|\psi(0)\rangle = |1\rangle$. What is $|\psi(t)\rangle$, the state of the ammonia molecule after some time t ? What is the probability of finding the ammonia molecule in each of its energy eigenstates? What is the probability of finding the nitrogen atom above or below the plane of the hydrogen atoms?

EP-78. Angular Momentum - For a physical situation in which there are only 2 possible values for the z-component of the angular momentum, it is often convenient to use the eigenstates of the \hat{L}_z operator as the basis for calculations. In this basis, the \hat{L}_z operator and its eigenstates take the following form:

$$\hat{L}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{with eigenstates} \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- (a) Let us define 2 new states using the \hat{L}_z eigenstates as a basis.

$$|A\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \quad |B\rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} -i \\ \sqrt{2} \end{pmatrix}$$

Are these two states normalized? Are these two states orthogonal?

- (b) Using the eigenstates of \hat{L}_z as a basis, the operator representing measurements of the y-component of the angular momentum is given as follows:

$$\hat{L}_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Find the eigenvalues and eigenstates of the \hat{L}_y operator in the \hat{L}_z basis.

- (c) Express the states $|A\rangle$ and $|B\rangle$ using the eigenstates of \hat{L}_y as a basis.
- (d) Do \hat{L}_z and \hat{L}_y commute? What is the significance of whether they commute or not?