Physics 6H   Freshman Seminar   Fall 2004   Assignment #11

Readings:

- Albert - Chapter 3
- Boccio - K Mesons
- Boccio - Locality, EPR, SGs and Slits
- Boccio - Bell Theorem Intro

This week we will study all aspects of the two-level K-meson system, namely, energy eigenvalues and eigenvectors, time evolution, particle oscillation. In addition we will look at neutrino oscillations. Finally, we will be our study of non-locality, EPR, Bell and such.

Everyone Problems:

EP-66 Neutrino Oscillations

Individual Problems:

EP-59 Interference
EP-60 More Interference
EP-63 K-Mesons
EP-64 K-Meson oscillations
EP-65 Find the phase angle
EP-70 Spin in Magnetic Field
EP-74 Compatibility

Presentations:

- K-Mesons
- 2-Particle States
- EPR #1
- Bell and Socks

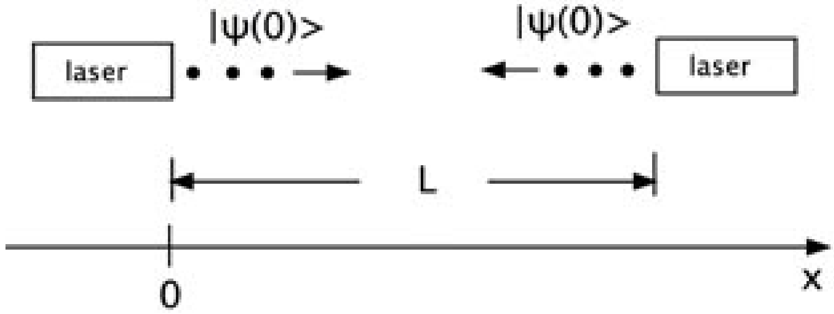
Seminar Break:

Extra Problems:

EP-59. Interference - Photons freely propagating through a vacuum have one value for their energy $E = h\nu$. This is therefore a 1-dimensional quantum mechanical system, and since the energy of a freely propagating photon does not change, it must be an eigenstate of the energy operator. So, if the state of the photon at $t = 0$ is denoted as $|\psi(0)\rangle$, then the eigenstate equation can be written $\hat{H}|\psi(0)\rangle = E|\psi(0)\rangle$. To see what happens to the state of the photon with time, we simply have to apply the time evolution operator

$$|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle = e^{-i\hat{H}t/\hbar}|\psi(0)\rangle = e^{-ih\nu t/\hbar}|\psi(0)\rangle = e^{-i2\pi\nu t}|\psi(0)\rangle = e^{-i2\pi x/\lambda}|\psi(0)\rangle$$

where the last expression uses the fact that $\nu = c/\lambda$ and that the distance it travels is $x = ct$. Notice that the relative probability of finding the photon at various points along the x-axis (the absolute probability depends on the number of photons emerging per unit time) does not change since the modulus-square of the factor in front of $|\psi(0)\rangle$ is 1. Consider the following situation. Two sources of identical photons face each other an emit photons at the same time. Let the distance between the two sources be L.



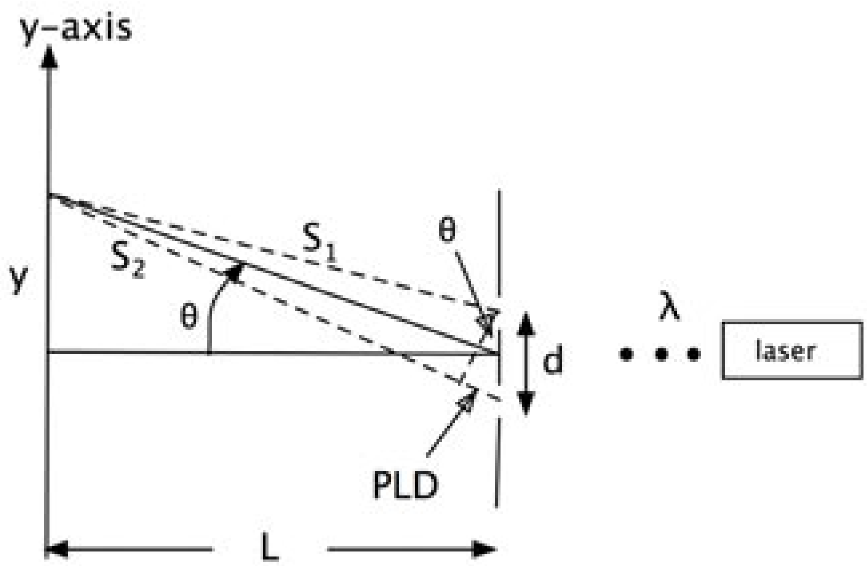
Notice that we are assuming the photons emerge from each source in state $|\psi(0)\rangle$. In between the two light sources we can detect photons but we do not know from which source they originated. Therefore, we have to treat the photons at a point along the x-axis as a superposition of the time-evolved state from the left source and the time-evolved state from the right source.

(a) What is this superposition state $|\psi(t)\rangle$ at a point x between the sources? Assume the photons have wavelength λ.

(b) Find the relative probability of detecting a photon at point x by evaluating $|\langle\psi(t)|\psi(t)\rangle|^2$ at the point x.

(c) Describe in words what your result is telling you. Does this correspond to anything you have seen when light is described as a wave?

EP-60. More Interference - Now let us tackle the two slit experiment with photons being shot at the slits one at a time. The situation looks something like the figure below. The distance between the slits, d is quite small (less than a mm) and the distance up the y-axis(screen) where the photons arrive is much,much less than L (the distance between the slits and the screen). In the figure, S_1 and S_2 are the lengths of the photon paths from the two slits to a point a distance y up the y-axis from the midpoint of the slits. The most important quantity is the difference in length between the two paths. The path length difference or PLD is shown in the figure.



We calculate PLD as follows:

$$PLD = d\sin\theta = d\left[\frac{y}{\left[L^2 + y^2\right]^{1/2}}\right] \approx \frac{yd}{L} \quad , \quad y << L$$

Show that the relative probability of detecting a photon at various points along the screen is approximately equal to $4\cos^2\left(\frac{\pi yd}{\lambda L}\right)$.

EP-63. K-Mesons - We discussed the K-mesons in the notes(see notes for background and details). The K^0 is made up of a down quark and a anti-strange quark, so its strangeness is +1 (S = +1). The $\bar{K}^0$ is made up of a anti-down quark and a strange quark, so its strangeness is -1 (S = 1). This information alone allows us to define the K^0 and $\bar{K}^0$ states like the hardness states and the strangeness operator like the harness operator.

$$\left|K^0\right\rangle = \begin{pmatrix}1\\0\end{pmatrix} \qquad \left|\bar{K}^0\right\rangle = \begin{pmatrix}0\\1\end{pmatrix} \qquad \hat{O}_S = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

Now comes the clincher. Neutral K-mesons never exist as K^0 or $\bar{K}^0$ for more than an infinitesimally small amount of time. Rather, they exist as a superposition of these two states.

$$\left|K_S\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \qquad \left|K_L\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

The subscript S stands for "short-lived" since this state has an average lifetime of $9\times10^{-11}\,\mathrm{sec}$ and the subscript L stands for "long-lived" since this state has an average lifetime of $5\times10^{-8}\,\mathrm{sec}$. It also turns out that $\left|K_S\right\rangle$ and $\left|K_L\right\rangle$ are energy eigenstates with the following energies

$$E_S = \left(m + \frac{\delta m}{2}\right)c^2 \qquad E_L = \left(m - \frac{\delta m}{2}\right)c^2$$

where m is the average mass of the particles represented by the $\left|K_S\right\rangle$ and $\left|K_L\right\rangle$ states and δm is the difference in mass between the particles represented by the $\left|K_S\right\rangle$ and $\left|K_L\right\rangle$ states. m is equal to about $497.7 MeV/c^2$ and δm is equal to about $2\times10^{-5} eV/c^2$.

Particle reactions produce neutral K-mesons of definite strangeness. So the neutral meson produced by a reaction must be a K^0 or a $\bar{K}^0$.

(a) Apply the time evolution operator to the state representing a K^0 (produced at $t=0$) and calculate the probability that this particle will still be a K^0 some time t later.
(b) Describe what is going on in words.
(c) Compare the time changes are taking place with the average lifetimes of the $\left|K_S\right\rangle$ and $\left|K_L\right\rangle$ states.
(d) Summarize what happens to a K^0 that is produced by a particle

reaction.

EP-64. K-Meson oscillations

An additional effect to worry about when thinking about the time development of theses states is that the $|K_L\rangle$ and $|K_S\rangle$ states decay with time. Thus, we expect that these states should have the time dependence

$$|K_L(t)\rangle = e^{-i\omega_L t - t/2\tau_L}|K_L\rangle \quad , \quad |K_S(t)\rangle = e^{-i\omega_S t - t/2\tau_S}|K_S\rangle$$

where

$$\omega_L = E_L/\hbar \quad , \quad E_L = \left(p^2c^2 + m_L^2c^4\right)^{1/2}$$

$$\omega_S = E_S/\hbar \quad , \quad E_S = \left(p^2c^2 + m_S^2c^4\right)^{1/2}$$

and

$$\tau_S \approx 0.9\times 10^{-10}\,\text{sec} \quad , \quad \tau_L \approx 560\times 10^{-10}\,\text{sec}$$

Suppose that a pure K_L beam is sent through a thin absorber whose only effect is to change the relative phase of the K_0 and $\overline{K}_0$ amplitudes by $10°$. Calculate the number of K_S decays, relative to the incident number of particles, that will be observed in the first 5 cm after the absorber. Assume the particles have momentum = mc.

EP-65. Find the phase angle

If CP is not conserved in the decay of neutral K mesons, then the states of definite energy are no longer the K_L, K_S states, but are slightly different states $|K'_L\rangle$ and $|K'_S\rangle$. One can write, for example,

$$|K'_L\rangle = (1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\overline{K}^0\rangle$$

where ε is a very small complex number $\left(|\varepsilon| \approx 2\times 10^{-3}\right)$ that is a measure of the lack of CP conservation in the decays. The amplitude for a particle to be in $|K'_L\rangle$ (or $|K'_S\rangle$) varies as $e^{-i\omega_L t - t/2\tau_L}\left(or \quad e^{-i\omega_S t - t/2\tau_S}\right)$ where

$$\hbar\omega_L = \left(p^2c^2 + m_L^2c^4\right)^{1/2}\left(or \quad \hbar\omega_S = \left(p^2c^2 + m_S^2c^4\right)^{1/2}\right)$$

and $\tau_L >> \tau_S$.

(a) Write out normalized expressions for the states $|K'_L\rangle$ and $|K'_S\rangle$ in terms of $|K_0\rangle$ and $|\overline{K}_0\rangle$.
(b) Calculate the ratio of the amplitude for a long-lived K to decay to two pions (a $CP = +1$ state) to the amplitude for a short-lived K to decay to two pions. What does a measurement of the ratio of these decay rates tell us about ε ?
(c) Suppose that a beam of purely long-lived K mesons is sent through an absorber whose only effect is to change the relative phase of the K_0 and $\overline{K}_0$ components by δ. Derive an expression for the number of two pion events observed as a function of time of travel from the absorber.

EP-66. Neutrino Oscillations

It is generally recognized that there are at least three different kinds of neutrinos. They can be distinguished by the reactions in which the neutrinos are created or absorbed. Let us call these three types of neutrino ν_e, ν_μ and ν_τ. It has been speculated that each of these neutrinos has a small but finite rest mass, possibly different for each type. Let us suppose, for this exam question, that there is a small perturbing interaction between these neutrino types, in the absence of which all three types of neutrinos have the same nonzero rest mass M_0 . The Hamiltonian of the system can be written as

$$\hat{H} = \hat{H}_0 + \hat{H}_1$$

where

$$\hat{H}_0 = \begin{pmatrix} M_0 & 0 & 0 \\ 0 & M_0 & 0 \\ 0 & 0 & M_0 \end{pmatrix} \rightarrow \text{no interactions present}$$

and

$$\hat{H}_1 = \begin{pmatrix} 0 & \hbar\omega_1 & \hbar\omega_1 \\ \hbar\omega_1 & 0 & \hbar\omega_1 \\ \hbar\omega_1 & \hbar\omega_1 & 0 \end{pmatrix} \rightarrow \text{effect of interactions}$$

where we have used the basis

$$|\nu_e\rangle = |1\rangle \quad , \quad |\nu_\mu\rangle = |2\rangle \quad , \quad |\nu_\tau\rangle = |3\rangle$$

(a) First assume that $\omega_1 = 0$, i.e., no interactions. What is the time development operator? Discuss what happens if the neutrino initially was in the state

$$|\psi(0)\rangle = |\nu_e\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ or } |\psi(0)\rangle = |\nu_\mu\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ or } |\psi(0)\rangle = |\nu_\tau\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

What is happening physically in this case ?

(b) Now assume that $\omega_1 \neq 0$, i.e., interactions are present. Also assume that at $t = 0$ the neutrino is in the state

$$|\psi(0)\rangle = |\nu_e\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

What is the probability as a function of time, that the neutrino will be in each of the other two states ?

(c) An experiment to detect the "neutrino oscillations" is being performed. The flight path of the neutrinos is 2000 meters. Their energy is 100 GeV. The sensitivity of the experiment is such that the presence of 1% of neutrinos different from those present at the start of the flight can be measured with confidence. Let $M_0 = 20\,eV$. What is the smallest value of $\hbar\omega_1$ that can be detected ? How does this depend on M_0? Don't ignore special relativity.

EP-70. Spin in Magnetic Field - Suppose that we have a spin = 1/2 particle interacting with a magnetic field via the Hamiltonian

$$\hat{H} = \begin{cases} -\vec{\mu}\cdot\vec{B} \quad , \quad \vec{B} = B\hat{e}_z \quad\quad 0 \le t < T \\ -\vec{\mu}\cdot\vec{B} \quad , \quad \vec{B} = B\hat{e}_y \quad\quad T \le t < 2T \end{cases}$$

where

$$\vec{\mu} = \mu_B \vec{\sigma}$$

and the system is in the initial($t=0$) state

$$|\psi(0)\rangle = |x+\rangle = \frac{1}{\sqrt{2}}\left(|z+\rangle + |z-\rangle\right)$$

Determine the probability that the state of the system at $t = 2T$ is

$$|\psi(2T)\rangle = |x+\rangle$$

in two ways:

(1) Using the Schrodinger equation (solving differential equations)
(2) Using the time development operator (using operator algebra)

EP-74. Compatibility - When the angular momentum along a direction in space of some atoms is measured, only two values result. Again, keeping things simple for measurements along the z-direction, we can define $\hat{O}_{L_z}$ and the eigenstates of $\hat{O}_{L_z}$ as usual.

$$\hat{O}_{L_z} = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ with } |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

In this basis, the operators for angular momentum along the x- and y-axes are as follows:

$$\hat{O}_{L_x} = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \hat{O}_{L_y} = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

(a) What are the eigenvalues and corresponding eigenstates of $\hat{O}_{L_x}$ and $\hat{O}_{L_y}$?
(b) What are the commutators of pairs of these operators?
(c) Are measurements of angular momentum along different direction in space compatible for these atoms?