**Quantum Theory of Measurement: The Infinite Regress**

How then does quantum theory describe the excitation of an atom by light? We might want to use this process to detect the location of a photon (by observed which atom get excited).

We imagine an atom, initially in the ground state that interacts with light.

What we find is that, rather than telling us about this transition, the Schrodinger equation merely predicts that atom went into a superposition state of being in both states.

$$|atom\rangle = \alpha_{gd}\left|atom_{ground}\right\rangle + \alpha_{ex}\left|atom_{excited}\right\rangle$$

The theory predicts that the coefficients(whose absolute squares represent transition probabilities) oscillate in time between 0 and 1 such that their sum always equals 1. This is all the theory has to say.

What is remarkable here is that a number of things have been left out of the discussion. For instance, if the atom began in the ground state and ended up in an excited state, it must have mad a transition between the two. But nowhere does the theory mention this transition. You might think that the transition occurs when the probability of being in the excited state reaches 1. But this is not so: the transition has a perfectly finite chance of having occurred long before this time. Indeed, it could have occurred at any time at all, other than $t = 0$. The theory is simply silent on this issue.

Furthermore, the transition is a vitally important event. After all, it is presumably the cause of the collapse process, or at least, strongly connected to it. But here too the theory is silent: you will have noticed that nowhere in this discussion have we even mentioned this collapse. It simply lies outside the range of issues treated by the Schrodinger equation.

We conclude that the collapse process occupies an anomalous position within quantum mechanics. It is required by the fact that observations occur, but it is not predicted by the quantum theory. That is why we had to add it as a postulate.

Before we go on, a caveat is necessary.

Some physicists hold the view that the state vector or wave function has no physical meaning at all. Rather then hold that the state vector should be understood as describing **our information about a system.** In this view the collapse process has no particular significance. Consider in this connection the question of whether it will rain at noon tomorrow. The weather bureau might predict that, say, there is a 30% chance of rain on the following day. Suppose we the wait until noon arrives, go outside and look. In the instant we become aware of the weather, the 30% probability "collapses" - to 100% if we find it to be raining, and to 0% if we find that it is not. No one is surprised by such behavior.

On the other hand, we should note that the large majority of physicists regard any change in the state vector as corresponding to a real physical process, as opposed to a change in our knowledge of that process.

So the Schrodinger equation does not describe the collapse of wave function caused by a measurement.

We then ask what does Schrodinger equation describe?

We find an astonishing result: according to Schrodinger equation

**measurements never happen**

Rather, what happens is infinite regress or a **quantum Zeno effect**.

In the preceding discussion we tried to measure the location of a photon by observing the transition of an atom to man excited state. What we found was that, rather than telling us about this transition, the Schrodinger equation merely predicted that the atom went into a superposition state.

The configuration we analyzed was a single atom in presence of a photon. But the configuration we should analyze is an entire collection of atoms — all atoms in the entire apparatus — together with photon.

To deal with this problem we need to write down a factor describing state of first atom, a second factor describing state of the second atom, and so on until we have covered every atom in apparatus — and then a final factor describing the photon.

If Schrodinger equation is applied to problem, it predicts a complex state.

The first term of this state consists of

- a factor describing atom #1 in superposition state
- factors describing all the other atoms in ground state
- a factor describing the photon at the location of atom #1

Similarly, the second term is

- a factor describing atom #2 in superposition state
- factors describing all the other atoms in ground state
- factor describing the photon at location of atom #2

and so forth.

Clearly, final state is an complex entanglement.

It is exceedingly difficult to work with.

So, for purposes of clarity, let us consider an even simpler problem — of measuring, not a photon's location, but its energy.

The advantage is that it allows us to work with detector that does consist of single atom, as opposed to apparatus that consists of many.

In fact, we consider an even simpler problem - a measurement whose sole function is to tell us whether a photon's energy is large or small.

The detection process is an excitation of a particular sort: **an ionization where an electron is removed from the atom**.

If the photon has high energy, the atom will be ionized by it; but if the photon's energy is low, the atom will not.

What does Schrodinger equation predict if it is applied to this problem? It predicts that

$$|system\rangle = a|photon_{low-energy}\rangle|atom_{neutral}\rangle + b|photon_{high-energy}\rangle|atom_{ionized}\rangle$$

where a and b are coefficients which give the probability amplitudes for each component.

This just expands the original superposition state.

All the dissatisfaction that we had with the earlier state applies as well to new result.

We set out to perform a measurement of photon's energy - but rather than doing this, the Schrodinger equation has informed us that a peculiar new state has developed, a state for which there is no classical analogue, and which seems to have nothing to do with a measurement.

But we need not stop here.

We could determine the photon's energy if we could find out whether the atom was ionized or not. So let us turn our attention to this problem.

An ionized atom possesses a net electric charge, and the charges respond to electric fields. So perhaps we can measure the photon's energy by applying an electric field to the atom and seeing what it does. If the photon had high energy, the atom will be deflected by field; if it had low energy, the atom will go straight.

What does the Schrodinger equation predict?

Perhaps you will not be surprised to learn that Schrodinger equation does not predict anything that we might interpret as a clean measurement.

Rather it predicts a yet larger entanglement – not an entanglement of two factors, but of three:

$$|system\rangle = a|photon_{low-energy}\rangle|atom_{neutral}\rangle|atom_{straight}\rangle + b|photon_{high-energy}\rangle|atom_{ionized}\rangle|atom_{deflects}\rangle$$

To determine whether the atom was deflected by the electric field, we can place a detector of atoms in undeflected beam. If this detector catches an atom, the photon had low energy; and if it does not, the atom had high energy. But, here too, all that develops is a yet larger entanglement:

$$|system\rangle = a|photon_{low-energy}\rangle|atom_{neutral}\rangle|atom_{straight}\rangle|\det ector_{catches\ atom}\rangle$$
$$+ b|photon_{high-energy}\rangle|atom_{ionized}\rangle|atom_{deflects}\rangle|\det ector_{does\ not\ catch\ atom}\rangle$$

The further we go into our analysis, the larger the entanglement grows. But, at no point does a measurement (collapse) occur.

Rather, what occurs is an infinite regress.

**Termination of the Infinite Regress: The Projection Postulate**

What would constitute a measurement of photon's energy?

Notice that in very last step of our analysis, an important transformation has occurred - we have included a macroscopic measuring device.

The puzzle presented to us by the quantum theory of measurement is that this device is predicted to be in an ambiguous state - a superposition.

But in reality, we know perfectly well that, if it is working correctly, a detector never behaves in such an obscure fashion.

It either detects the atom, or detects its absence.

In order to make this happen, in order to perform a measurement, we need to terminate the infinite regress at some stage, and replace the entanglement by a single term.

But this cannot be done by the Schrodinger equation.

It seems necessary, therefore, to divide time evolution of quantum system into two distinct parts. The first part lasts from the instant the quantum state is prepared until the moment just prior to the act of measurement.

During this period of time, the system evolves in a precisely specified manner according to time-dependent Schrodinger equation, in which forces acting on the system are included through potential energy function $V(\vec{r})$:

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\psi(0)\rangle = e^{-i\left(\frac{\vec{p}^2}{2m}+V(\vec{r})\right)t/\hbar}|\psi(0)\rangle$$

The result is an ever-increasing entanglement.

But act of measurement stands in sharp contrast to this orderly time development. No potential energy function and no energy operator exists for measurement, no equation analogous to Schrodinger equation describes its evolution in time.

Within orthodox quantum mechanics, measurement appears as an acausal process that, in a very real sense, seems to fall outside the theory.

Mathematician von Neumann described problem thoroughly in classic text Mathematical Foundations of Quantum Mechanics. From him we have inherited an **ad hoc** mathematical device to get around problem of infinite regress.

In place of the dynamical state, von Neumann described the **projection postulate**.

This is the postulate that, when a measurement occurs, entanglement is replaced by a single term.

The wave function of quantum system is **"projected onto"** the various possibilities provided by detector, with a probability of projection given by square of the coefficient multiplying each of the terms of the entanglement.

In context of example we have been considering, the projection postulate is that, at moment of measurement, state is collapsed either onto

$$\left|photon_{low-energy}\right\rangle\left|atom_{neutral}\right\rangle\left|atom_{straight}\right\rangle\left|\det ector_{catches\,atom}\right\rangle \quad \text{with probability } |a|^2$$

or onto

$$\left|photon_{high-energy}\right\rangle\left|atom_{ionized}\right\rangle\left|atom_{deflects}\right\rangle\left|\det ector_{does\,not\,catch\,atom}\right\rangle \quad \text{with probability } |b|^2$$

Notice that, in both states, detector is left in a perfectly well-defined state - and this is just what we mean by a measurement.

We emphasize that the projection postulate is nothing more than a mathematical statement of our postulate of the collapse of wave function.

We also emphasize that projection postulate does not correspond to a real, physical process. Rather it is a purely mathematical procedure, which gets us from the causal language of quantum mechanics to the experimental probabilities in a way that agrees with experiment.

In so doing, as Heisenberg put it, **what was probable becomes actual**.

But how does this occur? The answer is the subject of much contention.

The projection postulate presents us with a distasteful state of affairs. One dissatisfying feature is that we do not understand how the projection comes about. But a further dissatisfying element is that sometimes the postulate is not needed.

In certain situations, the postulate is required in order to make a measurement, but in other situations it is not!

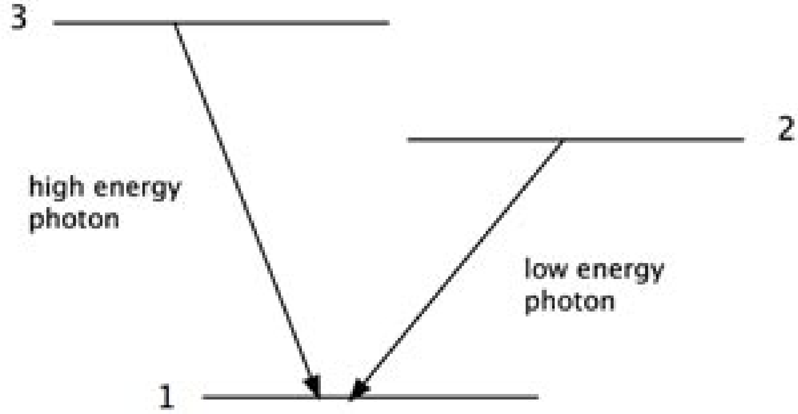
We will illustrate this point by returning to our above analysis of a photon whose energy can be either quite large or quite small.

We will discuss three situations.

In the first two, a measurement will occur quite naturally, within the framework of orderly evolution described by Schrodinger equation. Only in the third will Schrodinger equation prove to be insufficient, and the collapse postulate will be required.

How is such a photon produced?

It is produced by decay of an atom whose energy-level diagram is similar to that illustrated below.




The decay of such an atom from level 2 to level 1 produces a photon of low energy; that from 3 to 1 produces one of high energy. In case 1 we prepare a single atom in a well-defined state - either level 2 or level 3. Decay of this atom produces our photon. Via a series of experiments we have outlined above, its energy can be determined. If photon's energy is low, one state describes the final state; if it is high, other state does it.

This is very like a classical situation, in which the outcome of a measurement stands in direct correspondence with state of system prior to measurement. The initial state may not be known to us beforehand, but outcome of measurement reveals it to us. And finally, notice that at no stage have we been forced to invoke the projection postulate - the measurement has been effected solely as the result of the time-dependent Schrodinger equation.

In case 2 , we prepare a collection of such atoms, some in level 2 and some in level 3. Suppose for the sake of argument that 10% are in 2 and 90% in 3.

Such a collection is, of course, simply the mixture we have already encountered. Once again, we measure emitted photons, allowing sufficient time for entire ensemble of atoms to decay. Since each individual atom is described as in the previous case, the result of each measurement is perfectly straightforward, and once again the projection postulate is not required.

We will not be able to predict in advance which energy each particular photon will have, but this is no cause for concern. We simply do not have sufficient information - even in the classical universe, it is seldom possible to predict an event with this kind of certainty. But we do know that we will find 10% of the decay photons to be of low energy and 90% of high energy. Results like this lend support to our conventional understanding of reality as having an independent existence and well-defined attributes, regardless of the act of measurement.

Notice that, in both cases 1 and 2, the initial state of the atom or atoms corresponds directly to one of eigenstates of detection apparatus - a state where the detection apparatus has a definite value. As long as this is true, measurement in quantum mechanics does not contradict our naive view of reality.

However, as we have seen time and time again, the special feature of quantum mechanics is that it allows states other than simple eigenstates.

The situation changes dramatically if we consider them.

In case 3, we prepare a collection of atoms, each one of which is in a superposition state given by

$$|\phi\rangle = \sqrt{0.1}\left|atom_{in\,level\,2}\right\rangle + \sqrt{0.9}\left|atom_{in\,level\,3}\right\rangle$$

When such a state decays, the emitted photon is in a superposition state of its own:

$$|photon\rangle = \sqrt{0.1}\left|photon_{low}\right\rangle + \sqrt{0.9}\left|photon_{high}\right\rangle$$

If we now seek to measure the energy of this photon, the infinite regress encountered earlier will result - an infinite regress that can only be terminated by invoking the projection postulate.

Here lies the nub of the problem.

In case 3, the outcome of series of measurements we perform will be identical to that produced in case 2:

10% of photons will turn out to be of low energy

and

90% of high energy

In this case, however, we have no theoretical understanding of how the measurements came about.

Furthermore, what is it that distinguishes the pathological case 3 from the well-behaved cases 1 and 2? Only the choice of what initial state to be studied!

Those that happen to coincide with eigenstates of our measuring apparatus will not require the extra projection postulate.

Finally, notice that measuring devices exist whose eigenstates are not those of energy, but correspond to more complex superpositions of energy eigenstates – and if we substitute these devices for our previous ones, the need for projection postulate will evaporate.

Surely this is a most unsatisfactory state of affairs!