

QUANTUM MECHANICS AND THE COOKIE CUTTER PARADIGM

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Abstract

What has so far prevented us from decrypting quantum mechanics is the Cookie Cutter Paradigm, according to which the world's synchronic multiplicity derives from surfaces that carve up space in the manner of three-dimensional cookie cutters. This insidious notion is shown to be rooted in our neurophysiological make-up. An effort is made to liberate the physical world from this innate fallacy.

1 INTRODUCTION

We live in two worlds, each with its own kind of space, time, form, and substance. There is the *phenomenal* world, extended in phenomenal space and changing in phenomenal time, containing forms that at bottom are bounded regions filled with qualia (introspectible properties like red or green). And then there is the *physical* world, whose spatial aspect is given by the spatial relations that obtain among material objects, whose temporal aspect is given by the temporal relations that obtain among actual events and/or states of affairs, and whose forms are sets of possessed spatial relations—as will be shown in this article.

The first lesson of science is that appearances are deceptive. Things are not what they seem. It is a commonplace of the contemporary scientific world view that matter is intrinsically particulate even though phenomenal space is perfectly homogeneous. It is another commonplace that molecules are not bounded by colored surfaces even though all objects in phenomenal space are. We are sufficiently aware that the concept of “form” appropriate for phenomenal objects (a bounding surface) and the concept of “matter” that goes with it (continuous stuff filling bounded regions of phenomenal space) are not applicable to physical objects (objects in the physical world). But we are not yet sufficiently aware that the concept of “form” appropriate for physical objects and the concept of “matter” that goes with it, entail a

concept of “space” that is inconsistent with the standard mathematical description of physical space. This concept of “space”, moreover, entails a concept of “time” that is similarly incompatible with the standard mathematical description of physical time. Where physical time and space are concerned, we have yet to learn the principal lesson of science—appearances are deceptive.

This article serves three purposes. The first is to show that the seemingly insurmountable interpretational problems raised by quantum mechanics have their roots in ways of thinking about (and mathematically representing) space and time that are adequate for dealing with the phenomenal world, but that are as inconsistent with the ontological implications of quantum mechanics as the notion of absolute simultaneity is with special relativity. The second purpose is to arrive at appropriate ways of thinking about physical space and time and at adequate concepts of “form” and “substance”. The third purpose is to pinpoint the fallacy underlying the inappropriate concepts and ways of thinking—the Cookie Cutter Paradigm (CCP)—and to trace it to its neurophysiological roots. Because it has neurophysiological underpinnings and thus is, in a sense, innate, rejecting the CCP is no easy task. Yet it is worth the effort, for at the end we shall discover that the mathematical elegance and simplicity of quantum mechanics is matched by the depth and transparency of its ontological message.

Section 2 uses the paradigmatic two-slit experiment with electrons to argue that the reality of any set of spatial distinctions (i) depends on the system and (ii) is contingent on the existence of a matter of fact about the value of the corresponding position observable. What follows is that spatial distinctions cannot be regarded as inherent in space, and that, therefore, the standard, substantive, set-theoretic conception of space is the wrong conception of physical space.

Section 3 introduces a conception of space that is consistently applicable to the physical world. This portrays space as the totality of spatial relations existing among material objects. The corresponding concept of form is

analyzed and shown to imply the formlessness of all fundamental particles. The popular belief that a fundamental particle has the form of a point is thereby shown to be an illegitimate importation from the phenomenal world (in which all objects have forms for psychological if not neurophysiological reasons) into the world of physics.

Section 4 discusses two properties of phenomenal space—the quality of continuous extension and the absence of intrinsic divisions—and clarifies their relation to physical space. Two ways of thinking about space and form, one based on how things appear in phenomenal space and one appropriate for quantum mechanics, are diagrammatically represented and compared.

Section 5 deals with the tricky question of how to think correctly about the simplest forms that exist in the physical world—those that consist of a single relative position. This involves two kinds of conditional probability assignments (one noncounterfactual and one counterfactual) and two corresponding kinds of probability (one subjective and one objective).

Objective probabilities betoken an objective indefiniteness, and this entails that the spatial and temporal differentiation of the physical world has a finite limit. This is discussed in Sec. 6, wherein one also finds definitions of the adjective “macroscopic” as related to positions, objects, clocks, and space.

Section 7 discusses another paradigmatic experiment, the scattering of particles at right angles, and arrives at the conclusion that, *intrinsically*, all fundamental particles are identical in the strong sense of *numerical* identity. What makes this conclusion *seem* preposterous is the insidious notion that prevents us from decrypting quantum mechanics—the CCP.

Section 8 examines various definitions of “substance” in respect of their applicability to (i) a fundamental particle and (ii) that which each fundamental particle intrinsically is. It is found that there is exactly one substance—pure, unqualified existence,—and that the physical world arises from it by the only logically consistent expedient—the realization of spatial relations. By entering into spatial relations with itself, existence acquires the aspect of a multiplicity of entities. But what exists at either end of each spatial relation is identically the same entity. All there is, at bottom, is existence and spatial relations between existence and itself.

The final section is devoted to the CCP—the notion that the world’s synchronic multiplicity derives from surfaces that carve up space in the manner of three-dimensional cookie cutters. This is shown to be rooted in our neurophysiological make-up. Some of its consequences are examined to underline just how insidiously it prevents us from making sense of quantum mechanics.

2 THE CONTINGENT REALITY OF SPATIAL DISTINCTIONS

The two-slit experiment with electrons requires no introduction [1, 2]. According to Feynman [3], it “has in it the heart of quantum mechanics”. But this appears to be something so baffling that we are warned off speculating about its significance: “Do not keep saying to yourself, if you can possibly avoid it, ‘But how can it be like that?’ because you will go ‘down the drain’ into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that” [4]. The last word on the subject apparently is that “[e]lectrons seem to have modes of being, or modes of moving, available to them which are quite unlike what we know how to think about” [5].

As will be shown in this section, if “[n]obody knows how it can be like that”, it is because we use the wrong concept of space. If the behavior of electrons in two-slit experiments is “quite unlike what we know how to think about”, it is because we think about space as something that exists by itself—rather than by virtue of the spatial relations that obtain among the world’s material constituents—and that contains, as subsets or partitions, all conceivable spatial divisions [6, 7].

If all conceivable spatial divisions were intrinsic to physical space, they would have an unconditional reality (that is, they would be real for every physical object), and one of the following statements would necessarily be true of every physical object O contained in the union $R \cup R'$ of two disjoint spatial regions: O is inside R ; O is inside R' ; O has two parts, one inside R and one inside R' . If this were the case, no electron could ever pass through the union of two slits without passing through either slit in particular *and* without consisting of parts that pass through different slits. But this is precisely what electrons do when interference fringes are observed in two-slit experiments (and we do not postulate hidden variables).

What these interference fringes are trying to tell us is that spatial distinctions are not real *per se*. They have a conditional reality; they supervene—and thus depend—on the actual goings-on in the physical world. The conceptual distinction between two disjoint regions of space may be real for one physical object and nonexistent for another. If an electron can pass through the union of two slits without passing through either slit in particular and without consisting of parts that pass through different slits, it is because the distinction we make between the slits need not exist for the electron. The electron has not three *a priori* modes of going through the slits, as envisaged by Albert [5], but four:

- (i) it takes the first slit and not the second slit;
- (ii) it takes the second slit and not the first slit;
- (iii) it takes the first slit and it takes the second slit;
- (iv) it takes the “undivided union” of the slits.

The first three modes assign separate truth values to the two statements $\mathbf{e}_1 =$ “the electron takes the first slit” and $\mathbf{e}_2 =$ “the electron takes the second slit”. In so doing they entail that the distinction we mentally make between the regions defined by the slits, is real for the electron. (That no electron is ever found to move according to the third *a priori* mode betokens the electron’s indivisibility. If we find an electron taking the first slit and we also find an electron taking the second slit, we observe two electrons, rather than one.) The fourth mode of moving, overlooked by Albert, is available to the electron if that distinction has no reality for the electron, or if nothing in the physical world corresponds to the difference between \mathbf{e}_1 and \mathbf{e}_2 . Whereas (iii) is a conjunction of two propositions with inconsistent truth values, (iv) is a single proposition. The expression “undivided union” appears between quotation marks because, taken literally, it is self-contradictory: What is undivided cannot be a union. What the expression is intended to convey is that the division into separate regions of space, which exists in our minds, has no reality for the electron.

So when is this distinction real for the electron? The following is undisputed standard quantum mechanics: If something indicates the slit taken by the electron, or (equivalently) if there is a matter of fact about the slit taken by the electron, or (equivalently) if there is an actual event or state of affairs from which that slit can be inferred, then the electron has taken the indicated slit. In general: If something indicates the value taken by an observable Q then Q has taken the indicated value. The converse, too, is standard quantum mechanics, where “standard” connotes, in particular, the absence of hidden variables: If nothing indicates the slit taken by the electron, then the electron has not taken a particular slit. In general: If nothing indicates the value taken by an observable Q then Q has not taken any value whatsoever.

This (the converse) is not undisputed. There are “quantum realists” who deny the necessity of a property’s being indicated and instead affirm that probability one is sufficient for the possession of a property [8], or that an element of reality corresponding to an eigenvalue of an observable Q exists at a time t if the prior probability measure for the time t has the pure form $|\psi(t)\rangle\langle\psi(t)|$ and $|\psi(t)\rangle$ is an eigen-“state” of Q . But this is an error. As I have argued in detail [6, 7], the contingent properties of quantum-mechanical systems (or the values of quantum-mechanical observables) are *extrinsic*: they supervene on what happens or is the case in the rest of the world. To paraphrase Wheeler [9, 10], no property is a possessed property unless it is an indicated property [11].

If an observable Q has n eigenvalues q_i , a successful measurement of Q leads to an actual event or state of affairs that indicates n truth values, one for each proposition “ Q has the value q_i ”, $i = 1, \dots, n$. One of these truth values is “true”, the remaining $n - 1$ truth values are “false”. Any remotely feasible position measurement

permits us to distinguish between at most a finite number of spatial regions (although for theoretical purposes we may allow a countable set of distinguishable regions). In order to fully characterize a position measurement—and the observable being measured—we must specify the regions between which the measurement can distinguish.

As an illustration, let us consider a particle the support of whose wave function *for*—not *at* [12]—the time t is contained in R , and two arrays of detectors $\{D_i^1 | i = 1, \dots, n\}$ and $\{D_k^2 | k = 1, \dots, m\}$. The sensitive regions of each array of detectors are disjoint, and their union contains R . The two arrays thus measure different position observables on the particle, one capable of distinguishing between n regions and assigning truth values to the n propositions “The particle is inside the sensitive region R_i^1 of D_i^1 ”, the other capable of distinguishing between m regions and assigning truth values to m propositions. If the first position observable is (successfully) measured, n regions are distinct for the particle, and this finds expression in the existence of n truth values for the n propositions “The particle is inside R_i^1 ”.

If position measurements are seen in this light, it becomes clear that the necessary and sufficient condition for attributing a value to a position observable Q on a physical system O is also the necessary and sufficient condition for the reality, for O , of the distinctions we make between the regions in the spectrum of Q . Q has a value if and only if that value is indicated. But what is indicated is not just the value of Q . If Q is capable of distinguishing between n regions, n truth values are indicated: For each of the n regions, O ’s presence in it or absence from it can be inferred. But the distinction we make between spatial regions has a reality for O precisely if O ’s presence in or absence from each region can be separately affirmed. Hence the distinction we make between the n regions in the spectrum of Q has a reality for O if and only if the value of Q is indicated or, equivalently, if and only if Q has a value. Applied to the two-slit experiment with electrons this means that the distinction we make between the slits is real for an electron if and only if something indicates the slit taken by the electron, or (equivalently) if and only if the corresponding binary observable has a value, or (equivalently) if and only if both \mathbf{e}_1 and \mathbf{e}_2 possess truth values. If the propositions \mathbf{e}_1 and \mathbf{e}_2 are neither true nor false, they are meaningless, and the distinction we make between the slits has no reality for the electron.

The reality of spatial distinctions thus is contingent on what is, and what is not, indicated, and it also depends on the system. The standard, substantive, set-theoretic conception of space then cannot be the right conception of physical space. Physical space cannot be something that exists by itself, independently of the world’s material constituents, and that by itself contains, as subsets, all conceivable spatial divisions, for if it were such a thing,

the reality of spatial distinctions could not be contingent. All conceivable spatial distinctions would be real *per se*, and hence they would be real for all physical objects. The electron could not go through the “undivided union” of the slits, and no interference fringes could be observed.

3 SPACE AND FORM IN THE PHYSICAL WORLD

What, then, is the right way of thinking about physical space? One thing is clear: If space is not something that has intrinsic parts, the synchronic multiplicity of the physical world—the multiplicity of material objects [13] that exist at any one time—cannot be defined in terms of the “parts of space”. It can only be defined in terms of the material objects that exist at any one time and/or in terms of the spatial relations that exist among these objects.

The following is equally clear: If space is not something that exists by itself and has intrinsic parts, spatial relations cannot be attributed to its parts. Spatial relations exist to the extent that they are attributable to (pairs of) material objects. And since spatial relations is all it takes to specify the spatial aspect of the physical world, it is safe to affirm that physical space *is* the totality of spatial relations existing among the world’s material constituents [14]. Here is another way of saying this: In order to describe the physical world, no reference to “space” is needed. All we need to refer to is the *positions* of material objects. And since these are relatively defined, reference to the relative positions of material objects—or, synonymously, to the spatial relations existing among material objects—is sufficient.

It follows that there is no such thing as an empty physical space. A physical world without material constituents is a spaceless world. It also follows that there is no such thing as a physical space containing a *single* material object, for two reasons. First, physical space contains spatial relations rather than objects. Second, a physical world containing a single material object lacks spatial relations; so it too is a spaceless world. (Such a world, moreover, would be indistinguishable from the single object it contains. It takes a minimum of two material objects to create a world containing something other than itself, such as spatially related objects. A world containing a single object is like the clapping of one hand.)

For the same reason that a physical world containing a single material object is spaceless, a noncomposite object—one that lacks parts—is formless. If the wrong way of thinking about physical space—as a substance in which all conceivable spatial divisions inhere—were the right way of thinking about physical space, the parts of a material object O would be defined by surfaces partitioning the space “occupied” by O , and the form of O would be defined by the surface bounding this space. A

material object would have as many parts as the space it “occupies”, and a material object without parts would have the form of a point. But there is no substantive space in which spatial divisions inhere. Therefore there are no surfaces by which the parts or the form of a material object could be defined. The ultimate parts of a material object are defined by their mutual spatial relations, and the form of a material object *is* the totality of its internal spatial relations—the spatial relations between its parts. A material object has as many internal spatial relations as it has pairs of parts. A noncomposite object lacks internal spatial relations, and therefore it also lacks a form.

While the form of a material object O is the totality of its internal spatial relations, the position of O —considered not as an attribute but as an observable with contingently attributable values—essentially consists in O ’s external spatial relations—those between O or its material constituents and objects having no constituents in common with O . Two extreme cases are of special interest: (i) If O is the entire universe, there are no spatial relations external to O . Hence the universe as a whole lacks a position, which makes good sense. The form of the universe, on the other hand, being the set of its internal spatial relations, is identical with the set of all possessed spatial relations, which is space. (ii) If O is a fundamental (meaning, noncomposite) particle—according to the standard model of elementary particle physics, the quarks and the leptons are fundamental,—there are no spatial relations internal to O . Thus a fundamental particle has a position but it lacks a form.

This too makes good sense. Even on the inappropriate view of space as intrinsically and infinitely divided, the notion that a noncomposite object has a form would be redundant—as redundant as attributing positions to the “points of space”. (The “points of space” are not objects that *have* positions. They *are* positions. They are properties that are available for attribution to material objects.) Saying that an object has the form of a point would be the same as saying that it has no parts, or that all its spatial relations are external, or that, relative to any given reference object, exactly one position can be attributed to it. There would be no reason to characterize a fundamental particle as having a pointlike form *over and above* being a noncomposite object.

The reasons for the popular belief that a particle has both a position and a (pointlike) form, have nothing to do with physics. They are psychological. They are to be found in the phenomenal world—the way we *perceive* the physical world. In the phenomenal world, the existence of spatial relations presupposes the existence of forms—visual percepts—to which spatial relations are attributable. Forms are *phenomenally* prior to spatial relations. In the physical world the converse is true: The form of a material object being the totality of its internal spatial relations, spatial relations are *ontologically* prior

to forms. Again, in the phenomenal world positions are (or seem to be [15]) realized individually, by objects with (ideally pointlike) forms. In the physical world positions are not realized by position-marking forms. They are realized by means of external spatial relations, and if the corresponding relata lack internal spatial relations, they are formless.

In the days when an atom was still widely thought of as a miniature solar system, Werner Heisenberg, if I remember right, argued that if atoms are to explain what the phenomenal world looks like, they cannot look like anything in the phenomenal world—an insight we have yet to assimilate in full. In the phenomenal world, in which objects are first of all bundles of qualia, every object necessarily has a form. The same is true of a world that is modeled after the phenomenal world, in which the shapes of things are bounding surfaces and matter is a continuous stuff filling the bounded regions. We smile at these simple-minded notions, yet the idea that a fundamental particle has a (pointlike) form is what becomes of these notions in the limiting case in which the bounding surface shrinks to a point. The space-filling stuff then metamorphoses into a material object that has both a position and a pointlike form.

4 CONTINUITY AND DISCRETE-NESS IN THE PHYSICAL WORLD

If we look into *phenomenal* space—the only space we *can* “look into”,—we perceive, first, a *quality* that goes beyond quantitative determinations and, second, a *unity* that goes beyond the unity of a set, defined by Cantor [16] as “a Many that allows itself to be thought of as a One”.

The quality to which I refer is the continuous extension of phenomenal space. This is as much a qualitative feature of our visual percepts as is the sensation of turquoise that we experience when looking at a tropical lagoon. Locutions such as “relative position”, “spatial relation”, or “distance” connote this qualitative feature as much as they connote quantitative determinability. Distances possess, in addition to their (more or less precise) values, this qualitative aspect, and nobody lacking our pre-conceptual grasp of phenomenal space is in a position to know it, anymore than Mary, confined from birth to a black-and-white room, was in a position to know color [17].

To discern the unity I have in mind, consider the visual image of a finite line segment L . Let us label its end points with real numbers a and b . L is in an obvious sense divisible into smaller segments, but nothing in its image warrants the notion that it is intrinsically multiple, let alone that it is a concatenation of point individuals with the cardinality of the real numbers. While between the real numbers a and b there exists a nondenumerable

set S of real numbers, between the points labeled a and b there exists a perfectly continuous and intrinsically undifferentiated line segment L . (Note that the preposition “between” is used here in two distinct senses: a spatial sense—“between the points labeled a and b ”—and a non-spatial sense that signifies “greater than a and less than b ”.) S possesses something that L lacks, namely the multiplicity of a set of intrinsically distinct elements. And L possesses something that S lacks, namely the quality of continuous spatial extension and the unity that goes with it—the unity of something that exists in advance of divisions. To thinkers from Aristotle to Kant and Gauss it appeared self-evident that points on a line are extrinsic to the line, that they are added features not contained in it. They considered the line itself and, by implication, space itself is inherently undivided and as existing in an anterior relationship to limits and divisions. “Space is essentially one”, Kant wrote [18], “the manifold in it... arises entirely from the introduction of limits.”

Unfortunately we have become so accustomed to conflating the continuity of phenomenal space with the discreteness of the set \mathbb{R}^3 of triplets of real numbers that it is now almost impossible for us to tease them apart. We tend to visualize the reals as a continuous line without realizing that the very act of visualization introduces a qualitative element that is not warranted by the mathematical construction of the reals. Conversely, in our attempt to get a conceptual grip on physical space we seize on the reals as a set that appears to contain sufficiently many elements to “fill” a continuous line and thus to possess its continuity. We even deprive ourselves of words that are needed to distinguish between S and L , as when we apply the adjective “continuous” to a *set*.

I will not quarrel over mathematical practice, which is justified by its results. Nobody is forbidden to call a self-adjoint operator “elephant” and a spectral decomposition “trunk”, which makes it possible to prove a theorem according to which every elephant has a trunk. What is illegitimate is to create the impression that this theorem has something to do with biological pachyderms. By the same token, nobody is forbidden to call “continuous” a set that is cardinally equal to (or greater than) the real numbers. But it is illegitimate to identify this “continuity” with the continuity of a line in phenomenal space. It is therefore preferable to reserve the word “continuous” for the continuity that is a unique pre-theoretical feature of our visual percepts and images.

Continuity, so defined, is a feature of objects in phenomenal space—that is, of visual percepts,—and it is not a feature of any mathematical set. Is continuity a feature of objects in physical space? The answer is “Yes”, or at least “Why not?”. Physical space is the set of all possessed spatial relations, and to each of these relations we can consistently attribute the quality of undivided spatial continuity. Consider the distance $\mathcal{D}(AB)$ between two material objects A and B . We tend to think of $\mathcal{D}(AB)$

as a “quantity”, and we tend to attribute to it the multiplicity of a segment of the “real line”. That is, we tend to take it for granted that there are as many places between the two objects as there are real numbers between 0 and $d(AB)$, the value of $\mathcal{D}(AB)$, assuming that $\mathcal{D}(AB)$ has a value. In reality there are as many places in the physical world as there are material objects. The places at which objects may be located (that is, the relative positions that may be attributed to them) do not exist unless objects are located there (that is, unless there are objects that possess them). There is no physical multiplicity that could qualify as inherent in a single spatial relation. There aren’t any points or places between A and B unless other material objects are situated between A and B . It takes a third object C to introduce another two distances $\mathcal{D}(AC)$ and $\mathcal{D}(CB)$ such that $d(AB) = d(AC) + d(CB)$. The same equation does not hold among the three distances $\mathcal{D}(AB)$, $\mathcal{D}(AC)$ and $\mathcal{D}(CB)$. None of these distances is the sum of anything. $\mathcal{D}(AB)$ is not a quantity “made up” of quantities; it is a relation that possesses both the quality of undivided spatial extension and a value $d(AB)$. It interposes no places between A and B . If anything interposes a location between A and B , it is another material object C .

Thus Kant was wrong after all. What is “essentially one” is not space but *each spatial relation*. Being the totality of all possessed spatial relations, space has the multiplicity of the set of all pairs of material objects. Nor does “the manifold” in space, as he calls it, arise from the introduction of limits. Kant labored under the delusion of the CCP—the notion that the world’s synchronic multiplicity derives from surfaces that carve up space like three-dimensional cookie cutters,—as we all do, thanks to our neurophysiological make-up (see Sec. 9). In reality the synchronic multiplicity of the physical world is due to the existence of spatial relations.

Figure 1 summarizes and contrasts two ways of thinking about space and form. The naive view, based on how things appear in phenomenal space, is illustrated by Fig. 1A, while the view appropriate for quantum mechanics is illustrated by Fig. 1B. The three lines forming the big triangle in both figures symbolically represent the spatial relations existing between three material objects. On the naive view, every material object has a form, which is either a bounding surface or a point. The big circles in Fig. 1A represent bounding surfaces; the small black circle represents a pointlike form. On the correct view, the form of an object is a set of internal spatial relations, and a noncomposite object is formless. The small triangles in Fig. 1B symbolically represent forms consisting of spatial relations. The absence of a small triangle from the third corner of the big triangle in Fig. 1B indicates the formlessness of a fundamental particle. The square frame, finally, represents a space that exists independently of material objects and “contains” them. In Fig. 1B this frame is conspicuous by its absence. In the

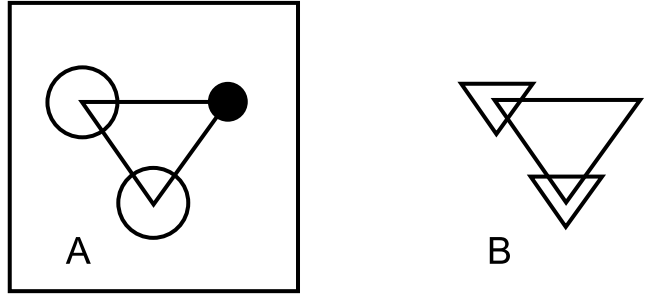


Figure 1: Two ways of thinking about space and form. A—the naive view based on how things appear in phenomenal space. B—the correct view. The three lines forming the big triangle in both figures represent the spatial relations between three material objects. The big circles represent forms that are bounding surfaces. The small black circle represents a pointlike form. The small triangles represent forms consisting of spatial relations. The absence of a small triangle from one of the corners of the big triangle in B indicates the formlessness of a noncomposite object. The square frame represents a space that exists independently of material objects and “contains” them. It is absent from B because physical space is nothing more than the totality of all possessed spatial relations.

physical world, there is no space other than (i) the set of all relative positions and (ii) the quality of continuous spatial extension that is possessed by each relative position. While spatial relations in the phenomenal world derive their spatial character from phenomenal space, physical space derives its spatial character from the relations that it contains (in the proper, set-theoretic sense of “containment”). There is no physical space *over and above* the spatial relations that make up the forms of all beasts and baubles in the physical world.

5 THE SHAPES OF SMALL THINGS

The non-visualizable character of the quantum world was recognized early on [19]. It led to the view that “[t]he right way to understand quantum mechanics is not as a true description of physical reality but rather as an *instrument* for predicting the outcomes of laboratory experiments. There is no coherent interpretation of the quantum-mechanical formalism as describing an unobservable reality that is responsible for those experimental results. That reality is forever beyond our ken.” [20] What is ultimately responsible for the agnosticism of this view is our apparent inability to think of the spatial aspect of the physical world except in the terms laid down by the CCP, according to which (i) the parts of a material object are defined by the parts of the space it “occupies” and (ii) the “parts of space” inhere in a space that exists

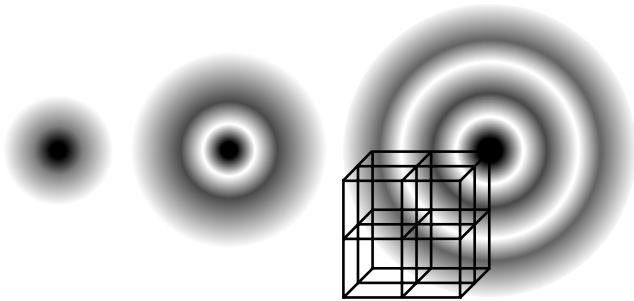


Figure 2: The forms of a hydrogen atom in its first three spherically symmetric “states”. Darker shades of gray represent higher “probability densities” (not to scale). Some nonexistent divisions of the third spherically symmetrical “state” are shown.

independently of the objects it “contains”. This way of thinking about space may be appropriate for dealing with ordinary sensory perceptions but, as Bohr realized back in 1923, it is not appropriate for dealing with atoms: “It is my personal opinion that these [interpretational] difficulties are of such a nature that they hardly allow us to hope that we shall be able, within the world of the atom, to carry through a description in space and time that corresponds to our ordinary sensory perceptions.” [21]

Does the inadequacy of the standard, substantive, set-theoretic conception of space imply the impossibility of arriving at *any* adequate conception of the world’s spatial aspect? If we subscribe to this *non sequitur* then we have to conclude with Stapp “that ‘space,’ like color, lies in the mind of the beholder” [22]. However, as the previous sections have shown, there *are* adequate ways of conceiving of the spatial aspect of the physical world. What “lies in the mind of the beholder” is the CCP and the concept of space it entails.

What is indeed strictly non-visualizable, is an individual fundamental particle: What does not have a form cannot be visualized. By no means, however, does this imply that we must resign ourselves to an agnostic attitude with respect to the real nature of a fundamental particle. Quite to the contrary, the formlessness of the fundamental constituents of matter tells us something of considerable ontological significance, for it implies, among other things, that fundamental particles exist in space *only* in the sense that they possess (more or less “fuzzy”) relative positions. Considered in itself (that is, out of relation to other material objects), a fundamental particle has neither a position nor a form. It has no position because its position consists in its relations to other material objects; and it has no form because it lacks internal relations. But an object that has neither a position nor a form cannot be said to exist in space. Physical space exists *between* the fundamental particles; it is *spanned* by their spatial relations.

The simplest thing that we *can* visualize—provided that we are clear about the significance of the various features of our visualization—is a single relative position. As an example, let us visualize a hydrogen atom in one of its spherically symmetric stationary “states” (Fig. 2). If we ignore the proton’s internal relative positions (or if instead we visualize a positronium atom) then all there is to be visualized is the position of the electron relative to the other particle (or vice versa). Since this is the atom’s only internal relative position, it constitutes its form. It is a most peculiar kind of form in that it has no parts. If it had parts, there would be as many positions relative to the other particle as there are parts, but there is only this one relative position.

Yet the cloudlike pictures in Fig. 2 do have parts, some of which are outlined for the third spherically symmetric “state”. What do these parts represent? And what do the varying densities of those clouds represent? One thing is clear: The parts are not features of any actual state of affairs; neither therefore are the local densities of the clouds. What the parts represent is something that is not the case but that would be the case if certain conditions were fulfilled. These are the conditions:

(i) There exists an array of detectors D_i with mutually disjoint sensitive regions R_i such that $\bigcup_i R_i$ contains the support of the wave function $\psi(\mathbf{r})$ associated with the electron’s position relative to the proton.

A detector, here, is anything capable of indicating the electron’s presence inside a particular region of the “space” \mathbb{R}^3 of precise values associated with the electron’s position relative to the proton (not to be confused with physical space).

(ii) Exactly one of the detectors clicks, and thereby indicates the electron’s presence in a particular region R_k .

If these conditions are fulfilled, the prior probability that the detector that clicks is the one with the sensitive region R_k , is given by

$$p(R_k) = \int_{R_k} d^3r |\psi(\mathbf{r})|^2.$$

What is represented by the cloud, accordingly, is the “probability density” $|\psi(\mathbf{r})|^2$. For two reasons this appellation is a misnomer. First, the probability that something happens in a particular region is not something that exists inside that region [12]. Second, and more importantly, if conditions (i) and (ii) are not fulfilled, the distinctions we make between the regions R_i have no reality for the atom’s internal relative position, as was shown in Sec. 2. In this case there are no regions to which any kind of content (including “probability content”) can be attributed. *A fortiori* there are no infinitesimal regions to which an infinitesimal content $d^3r |\psi(\mathbf{r})|^2$ can be attributed.

Figure 2 thus represents something that is *not* the case inasmuch as it represents *both* spatial divisions *and* prob-

abilities that are distributed over them. If the spatial divisions are real for the electron (that is, if they are real with regard to its position relative to the proton), the probabilities $p(R_i)$ cannot be regarded as representing an objective feature of the physical world, for in this case the correct probability of finding the electron in a particular region R_k is not $p(R_k)$ but either zero or one—the electron is inside one of these regions. (By “correct” I mean that the probability is assigned on the basis of all relevant facts, including the electron’s presence in a particular region. $p(R_i)$ is our best guess as long as we are ignorant of the particular region containing the electron.) And if the probabilities $p(R_i)$ do represent an objective feature of the physical world, the conceptual partition of the “space” \mathbb{R}^3 into the regions R_i does not.

In what sense can probability assignments represent an objective feature of the physical world? The answer hinges on the fact that quantum-mechanical probability assignments, which are always conditional on the existence of a matter of fact concerning the value of a specific observable at a specific time, have two valid readings. The first is noncounterfactual: If the distinctions we make between the regions R_i are real for the electron at a time t —the time at which one of the detectors clicks [23],— $p(R_i)$ is *subjective* in that it is assigned without taking account of the particular region that contains the electron at the time t . It reflects our prior ignorance of that region.

The second reading is counterfactual: If (contrary to fact) the distinctions between the regions R_i were real for the electron at a time t (that is, if there were a matter of fact about the particular region containing the electron at the time t), the electron would be inside the region R_i at the time t with an objective probability $p(R_i)$. In this case $p(R_i)$ is *objective* in the sense that it is assigned on the basis of all relevant facts. No fact that has a bearing on the probability of finding the electron inside R_i at the time t has been ignored—there is nothing for us to be ignorant of [24].

The laws of quantum mechanics thus allow us to assign not only subjective probabilities to the possible results of performed measurements but also objective probabilities to the possible results of unperformed measurements. Objective probability assignments apprise us of an objective indefiniteness in the physical world, and they do so in the only adequate language—the language of counterfactuals.

It has been said that, although the terminology of “indefinite values” is prevalent in some elementary textbooks, what is really intended is that a certain observable does not possess a value at all [25]. But there is more than that to the indefiniteness of the hydrogen atom’s internal relative position, which finds expression in assignments of objective probabilities to counterfactuals. What essentially is implied by this indefiniteness is that the physical world is only finitely differentiated. As this has been dis-

cussed in detail in Refs. [6] and [7], only a brief summary of the discussion will be given, for the sake of sketching a reasonably complete picture, in the following section.

6 THE SPATIOTEMPORAL DIFFERENTIATION OF THE PHYSICAL WORLD

Classical physics is consistent with the view that space is both intrinsically and infinitely differentiated. *Infinitely* because the position q of a material object always has a precise value. This is the same as saying that for every partition $\{R_i\}$ of space into mutually disjoint regions, however small, and for every region R_i , the proposition “ q is in R_i ” is either true or false. If we conceptually divide up space into infinitely many regions, there are infinitely many alternatives between which we can, in principle, distinguish. And *intrinsically* because this holds for *every* material object, which allows us to attribute the spatial divisions to “space itself”.

In reality space is neither intrinsically nor infinitely differentiated. Physical space is the totality of relative positions existing between material objects, and *none* of these relative positions has a precise value. For every relative position we can find a partition $\{R_i\}$ of the “space” \mathbb{R}^3 of precise values into sufficiently small but *finite* regions such that the corresponding position observable (cf. Sec. 2) *never* possesses a value. Thus one can never distinguish between more than a finite number of alternative regions, and this comes to saying that the physical world is only finitely differentiated spacewise.

However, while no relative position ever has a precise value, some relative positions are so “sharp” that their factually warranted values evolve in a completely predictable manner. Every indicated value (such as being inside region R_k) can be predicted, via the pertinent classical laws, on the basis of indicated earlier values. The indicated values of such relative positions evince no statistical variations; they are not *manifestly* fuzzy. It is therefore *quantitatively* correct (not only for all practical purposes but strictly) to treat these *macroscopic positions* as forming a self-contained system of causally connected, *intrinsic* positions—the classical domain—rather than a system of statistically correlated, *extrinsic* positions that presupposes position-indicating facts and, hence, the classical domain [11].

The sharp relative positions of classical physics “mesh”; they can be embedded in a point set cardinally equal to \mathbb{R}^3 , as spatial relations between points, and this point set can be identified with physical space. The fuzzy relative positions of quantum physics do not “mesh”; they cannot be attributed to the members of a point set. Each relative position comes with its own “space” \mathbb{R}^3 of precise values and potentially attributable regions, but the precise values and the attributable regions are both fig-

ments of our imagination. No actually existing detector has a sharply bounded (let alone pointlike) sensitive region.

Macroscopic positions lie somewhere in between. To take account of their fuzziness, they should not be embedded in a point set cardinally equal to \mathbb{R}^3 . But they can be embedded, as spatial relations, in a *macroscopic space* whose elements are fuzzy “points” or fuzzily bounded “regions”. Nothing needs to be said about the spacing of the “points” or the size of the “regions” except—

(i) No finite portion of macroscopic space contains more than a finite number of such “points” or “regions”. This is warranted by the fact that one can never distinguish between more than a finite number of alternative regions, or the fact that physical space is only finitely differentiated spacewise.

(ii) The “points” or elemental “regions” are sufficiently close or small to accommodate all macroscopic positions as relations between them.

The “points” or elemental “regions” being fuzzy, their relative positions are fuzzy. But if we let the “points” or elemental “regions” be sufficiently small or close, we can make their relative positions sharper than the macroscopic positions ever are. This allows us to represent all macroscopic positions as spatial relations between the “points” or elemental “regions” of macroscopic space without loss of factually warranted precision. Macroscopic positions “mesh” in the sense that the relative positions of macroscopic objects can be attributed to the elements of macroscopic space. The elements of macroscopic space can then be attributed as positions to macroscopic objects. (*Macroscopic objects* are objects with macroscopic relative positions.)

Since macroscopic positions are not manifestly fuzzy, neither are the fuzzy “points” or fuzzily bounded “regions” of macroscopic space. Fuzziness implies probability distributions, and there is nothing over which the fuzzy points or boundaries could be probabilistically distributed. There is no finer partition than the partition of macroscopic space into its elemental “regions”. Even this theoretical partition by definition exceeds the limit of the world’s actual spatial differentiation. The probabilities corresponding to any finer partition are therefore objective and distributed over counterfactuals. Our conceptual spatial distinctions, carried beyond this limit, bottom out in a “sea” of objective probabilities.

The elements of macroscopic space are none of the things we are neurophysiologically disposed to attribute to space (see Sec. 9). They are not mathematical points inasmuch as we can differentiate them, if only in our minds. They are not regions inasmuch as the sharpness of a region-defining boundary entails an unrealized degree of spatial differentiation. For the same reason they are not fuzzy “points” or fuzzily bounded “regions”. They are “regions” only in the sense that we can differentiate them in our minds, and they are “points” only in the

sense that they are not *actually* differentiated.

Many thinkers have been intrigued, and justifiably so, by the ability of the human mind to reproduce the physical world as faithfully as it does, or seems to do, depending on logic and mathematics alone [26, 27]. The success of physics is indeed astonishing, but the difficulties we experience in trying to understand quantum mechanics reveal that we are not all that well-equipped mentally. On the contrary, we seem to be neurophysiologically disposed to misunderstand quantum mechanics. The above attempt to characterize the *right* way of thinking about the actual extent of the world’s spatial differentiation is a case in point. It starts from a *wrong* way of thinking about the spatial aspect of the world that appears to be innate, and then proceeds by elimination (points that are unlike mathematical points, regions that are unlike mathematical regions. . .). If it is in fact innate then this is the best we can do.

What is true of the world’s spatial aspect is equally true of its temporal aspect. There is no such thing as an intrinsically differentiated time. What is temporally differentiated is physical systems, and every physical system is temporally differentiated to the extent that it passes through distinct successive states, in the proper sense of “state” connoting properties indicated by facts. Not only the positions of things but also the times at which they are possessed are extrinsic. While attributable positions are physically defined by the not manifestly fuzzy sensitive regions of macroscopic detectors, attributable times are physically defined by the not manifestly fuzzy times indicated by macroscopic clocks. (Since the sensitive regions of macroscopic detectors and the times indicated by macroscopic clocks can be treated as intrinsic, no vicious regress is implied.)

The finite temporal differentiation of any finite physical system is a direct consequence of the world’s finite spatial differentiation. Every physical system is temporally differentiated to the extent that it passes through distinct successive states, and no finite system passes through an infinite number of such states in a finite time span. During such a time span, a macroscopic clock, indicating time by means of some macroscopic position, can indicate no more than a finite number of distinct times, and therefore there exist no more than a finite number of such times. These times are both “instants” and “intervals” in the same sense that the elements of macroscopic space are both “points” and “regions”. They are fuzzy or fuzzily bounded, but only in relation to an imaginary background that is more differentiated time-wise than anything in the physical world.

7 FUNDAMENTAL PARTICLES

While the two-slit experiment with electrons can teach us how to think correctly about physical space and the shapes of material things, the following, equally paradigm-

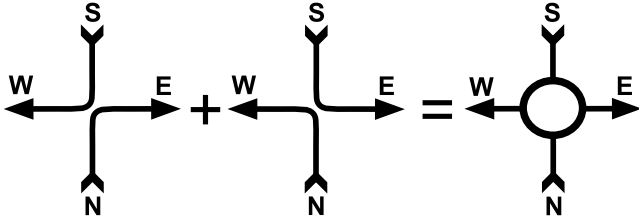


Figure 3: Scattering of two particles at right angles. If the alternatives on the left-hand side of this symbolic equation are experimentally distinguishable, the equation holds for probabilities. In this case the diagram on the right-hand side gives an incomplete picture of what actually happens. If the alternatives are indistinguishable experimentally, the equation holds for amplitudes. In this case the diagram on the right-hand side gives the complete picture while the diagrams on the left-hand side are overspecific: They involve a distinction that Nature does not make.

matic (and equally enigmatic) scattering experiment can teach us how to think correctly about the substance aspect of the physical world. We will consider the special case in which there are two incoming particles, one (N) moving northwards and one (S) moving southwards, and two outgoing particles, one (E) moving eastwards and one (W) moving westwards. The assumed final state (one particle eastbound and one particle westbound) can be reached in either of two ways (Fig. 3).

It is a fundamental principle of quantum mechanics that the probability of a process \mathcal{P} capable of following several alternatives depends on whether or not the alternative taken by the process is indicated or capable of being indicated. If something actually indicates the alternative taken, the probability of the process is given by the sum of the probabilities associated with its alternatives. By saying that the alternative taken by \mathcal{P} is capable of being indicated I mean that the alternatives of \mathcal{P} are correlated with the alternatives of another process \mathcal{P}' such that a determination of the alternative taken by \mathcal{P}' reveals the alternative taken by \mathcal{P} . In this case, too, the probability of \mathcal{P} is given by the sum of the probabilities associated with its alternatives. Paradigm examples of such a situation are the experiments of Einstein, Podolsky, and Rosen [28, 29] and of Englert, Scully, and Walther (ESW) [30, 31, 32]. On the other hand, if nothing indicates or is capable of indicating the alternative taken by a process, the probability of the process is given by the absolute square of the sum of the amplitudes associated with its alternatives [33].

Thus if there is a matter of fact about the slit taken by the electron, we add the probabilities associated with the alternatives e_1 and e_2 , and if nothing indicates (or is capable of indicating) the slit taken by the electron, we add the amplitudes associated with these alternatives.

By the same token, if there is a matter of fact about the alternative taken by the above scattering process, the probability of the process is given by

$$p_e(E, W) = |\langle EW|NS\rangle|^2 + |\langle WE|NS\rangle|^2,$$

and if nothing indicates the alternative taken, the probability of the process is given by

$$p_i(E, W) = |\langle EW|NS\rangle + \langle WE|NS\rangle|^2.$$

The subscripts e and i stand for “exclusive” and “interfering”, respectively. $\langle EW|NS\rangle$ and $\langle WE|NS\rangle$ are the respective probability amplitudes associated with the alternatives ($N \rightarrow E, S \rightarrow W$) and ($N \rightarrow W, S \rightarrow E$). For bosons $\langle EW|NS\rangle = \langle WE|NS\rangle$, so $p_b(E, W)$ is twice as large as $p_e(E, W)$:

$$p_b(E, W) = |2\langle EW|NS\rangle|^2 = 4|\langle EW|NS\rangle|^2 = 2p_e(E, W).$$

For fermions $\langle EW|NS\rangle = -\langle WE|NS\rangle$, so $p_f(E, W) = 0$. Both results are inconsistent with the notion that the scattering process actually follows *either* of the alternatives shown in Fig. 3. If there is a matter of fact about the alternative taken (for instance, if the two particles are not of the same type and there is no possibility of type swapping), we are entitled to assume that the actual process follows either alternative, which leads to $p_e(E, W)$. If there isn’t any matter of fact about the alternative taken, we cannot make this assumption, inasmuch it implies that the probability for scattering at right angles is given by $p_e(E, W)$, rather than by $p_b(E, W)$ or $p_f(E, W)$.

Once again we are confronted with a distinction that Nature does not make. If nothing indicates the alternative taken by the scattering process (which can only be the case if the two particles are of the same type), the distinction we make between the alternatives corresponds to nothing in the actual world. The process follows both alternatives, so we add amplitudes. Conversely, if something indicates the alternative taken by the scattering process (such as when the two particles are of distinct types), the distinction is real. The process follows either alternative, so we add probabilities.

The distinction we make between the two alternatives in Fig. 3 is based on the assumption of *transtemporal identity*: We assume that the individual particles possess permanent identities. In those cases in which the distinction cannot be made (or if it is made, must be rescinded by adding amplitudes), that assumption cannot be made: The particles do not possess permanent identities. Hence they do not possess the property of “being this very particle”, known to philosophers as “thisness” or “haecceity”. It follows that in those cases in which $p_e(E, W)$ is the correct probability, this is because the particles possess distinguishing characteristics and *not* because they possess something that would permanently individuate them even in the absence of distinguishing characteristics.

What, then, is the right way of thinking about two particles that lack distinguishing characteristics (and that therefore cannot be thought of as individuals that remain self-identical through change)? The two incoming particles as well as the two outgoing particles possess distinguishing characteristics: They travel in opposite directions, and they are in different places relative to the laboratory frame. But at the time of scattering no such distinguishing characteristics exist. How many things, then, exist at the time of scattering? According to the Identity of Indiscernibles, a principle of analytic ontology, there cannot be two absolutely indistinguishable things. Seen from the laboratory frame, the two particles are absolutely indistinguishable at the time of scattering, so on that account the answer should be: Only one thing exists at the time of scattering. Even at this time, however, the particles are in possession of a (more or less fuzzy) relative position, and this is sufficient for them to be two things even at the time of scattering.

And what is the right way of thinking about two indistinguishable particles *in themselves* (that is, out of *relation* to each other)? Since there is no such property as “thisness”, the existence of a (more or less fuzzy) relative position is not only sufficient but also necessary for two indistinguishable particles to be two. Apart from their relative position, there is nothing that could make them two. Hence *intrinsically* (out of relation to each other) the “two particles” are *not* two; they are identical, and this not in the weak sense of exact similarity but in the strong sense of *numerical identity*.

A preposterous conclusion! What makes it *seem* preposterous, however, is, at bottom, nothing but the CCP, for this tricks us into believing that the parts of a material object are defined by the parts of the space it “occupies”. If this were true of the physical world, no two physical objects could “occupy” the same space. *A fortiori*, all physical objects would be intrinsically distinct for the same reason that, according to the CCP, all conceptually distinct regions of space are intrinsically distinct (see Sec. 9).

If type conversions are allowed, the conclusion that particles of the same type, considered in themselves, are numerically identical, can be extended to particles of different types. Suppose that the two particles in our scattering experiment are of different types (say, a proton and a neutron in, a proton and a neutron out), but that particles of the first type can be converted into particles of the second type (e.g., a proton into a neutron) and vice versa. Further suppose that N and E are of type 1, and that S and W are of type 2. Then the probability of this scattering event is given by

$$p_i(E_1, W_2) = |\langle E_1 W_2 | N_1 S_2 \rangle + \langle W_2 E_1 | N_1 S_2 \rangle|^2,$$

where the indices specify the types to which the incoming and outgoing particles belong. Once again it is impossible

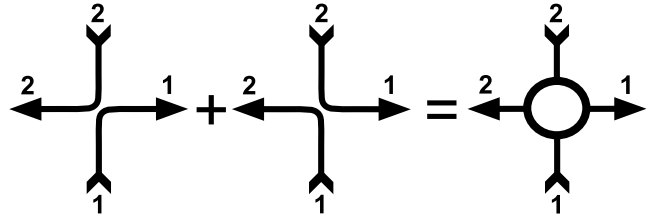


Figure 4: Same as Fig. 3, except for the possibility of type swapping. Since the alternatives on the left-hand side (with and without type swapping) are indistinguishable experimentally, the diagram on the right-hand side gives the complete story, while the diagrams on the left-hand side are again overspecific; they involve a distinction Nature does not make.

to say whether a particular incoming particle is the same as or different from a particular outgoing particle, for the distinction that we make between the alternatives in Fig. 4 is a distinction that Nature does not make. It exists solely in our minds.

The same argument that took us from $p_i(E, W)$ to the numerical identity of particles of the same type (considered in themselves), now takes us from $p(E_1, W_2)$ to the numerical identity of all particles of the same *basic* type (considered in themselves). What is characteristic of a basic particle species is that its members cannot be converted into members of a different basic species. How many basic particle species exist depends on the theory. According to the standard model, a member of one of the two species known respectively as hadrons and leptons cannot be converted into a member of the other species (the same applies to bosons and fermions), while in the so-called grand unified theories hadrons and leptons are mutually convertible, and in supersymmetric theories “once a fermion, always a fermion” is no longer true either. In these theories, being a hadron, lepton, boson, or fermion is an accidental property of something that by itself is neither hadron nor lepton nor boson nor fermion; there exists just one basic particle species. But whether or not the final theory (assuming that there will be one) permits conversions between all particle types, the property of belonging to a particular type can be regarded as accidental or contingent, and all existing fundamental particles can be regarded as being *intrinsically* identical in the strong sense of numerical identity. What is more, they ought to be so regarded, for the parameters that characterize a particle species (mass, spin, and charges) tell us how a particle behaves *in relation* to other particles (specifically, how its external relative positions at different times are statistically correlated), but they tell us nothing about the particle’s intrinsic behavior (that is, how it behaves *out of relation* to other particles).

8 SUBSTANCE AND THE PHYSICAL WORLD

What, then, is this mysterious thing \mathcal{X} that all particles intrinsically are, and what is a particle, given its intrinsic numerical identity with every other particle? The least inappropriate concept is “substance”. But this concept is laden with connotations that are inapplicable to the fundamental constituents of matter and must therefore be “peeled off”. The only positive characterization we get from Aristotle, who introduced the concept, is that a substance remains self-identical through change, and this is precisely what cannot be said of a particle, although it may be said of that which all particles intrinsically are. For the rest Aristotle gives us negative definitions: Substance is neither predicable (“sayable”) of anything nor present in anything as an aspect or property of it. This too is something we may hold on to as a characterization of \mathcal{X} . As a characterization of a particle it won’t do, for a particle may be said to be an instance or an aspect of \mathcal{X} .

For Locke, substance is that part of an individual thing in which its properties inhere. This concept of “substance” is nothing but an imaginary hook from which the properties are supposed to hang [34], a metaphysical glue that supposedly sticks them together. It may nevertheless be useful as a (partial) characterization of \mathcal{X} , for we may say that it is by virtue of \mathcal{X} that particles exist in the same world: \mathcal{X} is what ties them together as members of a single ontology [35]. But it is definitely useless as an intrinsic characterization of a fundamental particle, for, in itself, this *has no properties that need to be tied together*. Recall that not only its contingent properties (its external relative positions, the corresponding relative momenta, and its spin components relative to any given axis) but also its dynamical parameters (mass, spin, and charges) tell us nothing but how the particle behaves *in relation to* the rest of the world.

Substance has been conceived again as that which is capable of existing independently of anything else. By this definition, too, \mathcal{X} can be considered a substance, while an individual particle cannot. The existence of each fundamental particle depends on that which it intrinsically is, and the existence of a multiplicity of fundamental particles depends on the existence of spatial relations.

Finally, there is the ordinary sense of “substance”—what a thing is made of. This corresponds to Aristotle’s understanding of individual things as composites of matter and form, form being a bounding surface, and matter being the extended stuff that fills it. In Sec. 3 we have found the notion that a fundamental particle is a pointlike object to be a limiting case of this naive view: If the bounding surface shrinks to a point, what remains is a bit of stuff with a pointlike form. In reality there is no pointlike form; *a fortiori* there is no stuff that has one. What we *can* say of a fundamental particle is that it is

“made of” \mathcal{X} , but only in the sense that intrinsically it *is* \mathcal{X} . In this sense, \mathcal{X} is the substance of every fundamental particle, and every material object is made of \mathcal{X} plus the spatial relations that constitute its form. This contrast sharply with the Aristotelian understanding that every material object is made of spatially extended stuff and a boundary that constitutes its form.

We get a better grip on the fundamental constituents of matter if we resolve the following apparent contradiction. (i) Intrinsically (considered out of relation to other things), each fundamental particle is \mathcal{X} . (ii) Intrinsically, a fundamental particle lacks properties: Divested of its dynamical parameters and its contingent properties, it is nothing. How can a particle be both something (namely \mathcal{X}) and nothing? The contradiction is spurious, for statement (i) is about an *existing* fundamental particle (an actual ingredient of the physical world), while statement (ii) is about the *concept* of a fundamental particle (something that may or may not exist). By considering an existing particle out of relation to other things, statement (i) divests this particle of its dynamical parameters and its contingent properties. Statement (ii) further divests it of its *existence* as an actual ingredient of the physical world. The fact that in this case nothing remains, shows that the only thing that can be said of an existing fundamental particle, considered in itself, is that it exists. And this shows that all that \mathcal{X} bestows on an existing fundamental particle is *existence*.

If fundamental particles exist, there exist spatial relations, and if spatial relations exist, there exist material objects that either are fundamental particles or possess forms consisting of spatial relations between fundamental particles. We are inclined to think that the relations exist because the particles exist, but the opposite is at least equally true: The ultimate relata exist because the relations exist. The fundamental particles *qua instances* of \mathcal{X} owe their existence to the spatial relations that exist between them. Without these relations there would be no individual things; there would only be \mathcal{X} . The relations, on the other hand, owe their existence not to the relata but to that which they instantiate—to \mathcal{X} , which takes on the aspect of a multiplicity of relata because of the existence of relations.

I have just introduced a new concept: the (logical, rather than physical) process of *instantiation*. Instantiation is traditionally conceived as running parallel to predication: What gets instantiated is a predicable universal (an Aristotelian secondary substance or a Platonic Form like Horseness), and the resulting instance (e.g., a horse) is an impredicable individual. This way of thinking suggests that what is responsible for the instantiation is something that is present in the individual but absent from the universal, and this idea is at the root of the Platonic-Aristotelian dualism of Matter and Form and its subsequent transformations, including the erroneous idea that physical qualities are instantiated by the “points of

space” [36].

The process of instantiation that takes us from \mathcal{X} to the fundamental particles of matter is something else altogether. As we have just concluded, all that \mathcal{X} bestows on an existing fundamental particle is *existence*. Hence all that gets instantiated by the spatial relations is existence. \mathcal{X} is existence pure and simple. Instead of arriving at this conclusion by analyzing experimental data, we could postulate pure existence and ask ourselves how it comes to be instantiated. To be effective, the instantiator must exist in advance of the instantiation. But in advance of the instantiation of existence there is nothing but existence. (If there existed anything else, it would be an instance of existence.) Hence only existence can instantiate existence. But existence can instantiate itself solely by entering into relations with itself, for this is the only process consistent with the proper logical dependencies: The instances of existence exist because the instantiating relations exist; the instantiating relations exist because \mathcal{X} enters into relations with itself; and \mathcal{X} exists because it is “the one independent reality of which all things are an expression”—a dictionary definition of “the absolute” [37].

Thus if we adopt the premise of Parmenides that ultimately there exists a One Being [38]—which is also the first assumption common to all his predecessors,—and if we avoid the pitfalls created by the CCP, we can see how the physical world arises from that One Being by the only logically consistent expedient—the realization of spatial relations. The relations account for the existence of material forms, and the intrinsic numerical identity of the relata accounts for the behavior of indistinguishable particles. This is sufficient reason to postulate something like an absolute or pure existence as a fundamental explanatory principle within physics itself. In good Spinozistic tradition (but with what seems to me a great deal more justification) I submit that there exists exactly one substance, that every fundamental particle, considered in itself, is this one substance, and that the particles, considered in relation to each other, are simply the relata that are logically entailed by the existence of spatial relations *between this one substance and itself*. By entering into spatial relations with itself, \mathcal{X} acquires the aspect of a multiplicity of entities, all of which are intrinsically as indeterminate as \mathcal{X} itself because intrinsically each of them is \mathcal{X} . What exists at either end of the spatial relation of any pair of fundamental particles is identically the same \mathcal{X} . All there is, at bottom, is \mathcal{X} and spatial relations between \mathcal{X} and \mathcal{X} .

Spatial relations, as we have seen in Sec. 3, constitute both physical space and the shapes of material things. They are, moreover, *internal to \mathcal{X}* . Recall from Sec. 5 that a fundamental particle, considered in itself, does not exist in space; instead, space is a web of relations spun *between* the fundamental particles. Add to this the conclusion that all fundamental particles are identically the

same substance, and it follows that all spatial relations—and hence physical space itself—is internal to this one substance. The contrast with the vulgar conception of physical space as a container containing a multitude of self-existent substances (forms filled with stuff or point-like bits of stuff) could hardly be greater.

According to Russell [34], “substance” is a metaphysical mistake, due to transference to the world-structure of the structure of sentences composed of a subject and a predicate. Is this criticisms applicable to the one substance that I postulate as a fundamental explanatory principle? The answer is No, and the reason is that this postulate explains something that would otherwise remain unexplained. “No acceptable explanation for the miraculous identity of particles of the same type has ever been put forward”, Misner et al. [39] wrote. “That identity must be regarded, not as a triviality, but as a central mystery of physics.” The “miraculous identity of particles of the same type” is exemplified by the scattering experiment discussed in Sec. 7. This is sufficiently explained by the *numerical* identity of all fundamental particles, considered in themselves. If we approach the experimental data with the assumption of transtemporal identity, as we are inclined to do, nothing can possibly explain the observed statistics of that experiment. But if we can bring ourselves to admit that identically the same substance exists at either end of each spatial relation, that statistics becomes a natural consequence of this numerical identity.

The real problem is not identity—be it the identity of this particle with that particle or the identity of this region here with that region there. The real problem is difference. Identity being the fundamental truth about the physical world, it is the proper starting point for all physical explanations. What needs explaining is not how two particles or two regions of space can be identical but how they become effectively distinct. No theory explains everything. Aristotle explained motion but did not explain rest. He considered it “natural” for bodies to be at rest. Newton explained acceleration but did not explain uniform motion. He considered it “natural” for bodies to be in uniform motion. Einstein explained geodesic deviation but did not explain geodesic motion. He considered it “natural” for bodies to move along timelike geodesics. Quantum mechanics explains geodesic motion in terms of interference—in the classical limit all trajectories but one interfere destructively—but leaves unexplained the nature and existence of quantum-mechanical interference. One day this too may be explained [40]. But there is one question that no theory can aspire to answer, and this is the question of how is it that there is anything, rather than nothing. I would therefore consider truly fundamental only that theory which postulates nothing but pure, unqualified existence, and which can tell us how the physical world is both based on this existence (all particles are “made of” it) and suspended within it (space itself

is internal to it). That theory is quantum mechanics. It tells us that the physical world arises from pure existence by a self-differentiation that involves nothing but the realization of spatial relations. These account for all the differences that we observe in the physical world at any one time—differences between locations and differences between things.

No physical theory can define what essentially distinguishes the actual world from a nomologically possible world (any world consistent with the laws of physics). *A fortiori*, no physical theory can account for the actuality of the actual world or the factuality of facts. This is something that classical physics did not have to worry about, for classical theories never dealt with the actual world. Classical physics concerned nomologically possible worlds, and the question as to which of these worlds describes the actual world was to be settled by observation rather than by any theory.

In quantum physics, by contrast, possibilities and facts are inextricably entwined. Quantum physics assigns probabilities to the possible results of possible measurements on the basis of the actual results of actual measurements. The theory talks about what *actually* happens or is the case. It distinguishes between (i) alternatives all of which are mere *possibilities* (in which case it instructs us to add amplitudes) and (ii) alternatives of which one is *factual* (in which case it instructs us to add probabilities). While classical physics is indifferent to the mystery of existence, quantum physics confronts us with it, and in two ways. For one, it presents us with property-indicating *facts* that are uncaused and therefore intrinsically inexplicable [6]. In addition to that, it deals with fundamental particles, which in themselves are *existence* pure and simple. These numinous *apperçus* of “bare reality” play distinct ontological rôles. While the uncaused facts are the ultimate reason why there is something sayable, that which all particles intrinsically are is the ultimate reason why there is something of which anything can be said.

9 THE COOKIE CUTTER PARADIGM

What ultimately prevents us from decrypting quantum mechanics (without extraneous additions like hidden variables [41] or spontaneous collapses [42, 43, 44], without using “world” in the plural [45], and without distinguishing between a mind-constructed “internal” or “empirical” reality and a mind-independent “external” or “veiled” reality [46, 47] or bringing in consciousness in other ways [5, 48, 49, 50, 51, 52, 53, 54]) is the CCP—the innate idea that the world’s synchronic multiplicity derives from surfaces that divide up space like three-dimensional cookie cutters. The present section begins by explaining why this idea can be regarded as innate, and then focuses on how it prevents us from making sense

of quantum mechanics.

We are adept at recognizing three-dimensional objects in drawings containing only outlines. (In fact, we can’t help but perceive three-dimensional objects. We always see a Necker cube as pointing either in or out.) The reason why outlines are so easily recognized as objects lies in the manner in which the brain processes visual information.

The seminal work of Hubel and Wiesel [55] supports the following account. Visual information flows from retinal receptor cells via retinal ganglion cells to either of two lateral geniculate nuclei, and on to the primary visual cortex. The receptive field of each retinal ganglion or geniculate cell is divided into either an excitatory center and a concentric inhibitory surround (the “on center” configuration) or the reverse configuration (“off center”). (The group of retinal receptor cells from which a retinal ganglion or geniculate cell receives input is known as the cell’s receptive field.) Thus an “on center” cell responds best to a circular spot of light of a specific size, responds well to a bright line that just covers the center (since then most of the surround is not covered by the line), and does not respond at all if both center and surround are fully and equally illuminated.

When visual information reaches the visual cortex, two major transformations take place. One leads to the fusion of input from both eyes, the other to a rearrangement of incoming information so that most of its cells respond to *specifically oriented line segments*. The optimal stimulus may be a bright line on a dark background or the reverse, or it may be a boundary between light and dark regions. One group of orientation-specific neurons responds best to lines with just the right tilt in a particular part of the visual field. Another group of neurons, receiving input from the first group, is less particular about the position of the line and responds best if the line is swept in a particular direction across the visual field.

These data suggest that visual representations arise by way of an analysis of the visual field that is based on *contrast information from boundaries between homogeneously lit regions*. Data arriving from homogeneously colored and evenly lit regions do not make it into conscious awareness. The interior of such a region is *filled in* on the basis of contrast information stemming from its *boundary*. This interpretation receives strong support from the remarkable faithfulness of color perception to the reflectances of colored surfaces, and its corresponding insensibility to the actual spectral composition of the radiances of such surfaces. (If color perception is based on discontinuous color changes across edges, continuous variations in illumination across the visual field go unperceived [56].) It also explains why the blind spot is not perceived if it falls inside a homogeneous region (no sense data arrive from such a region anyway), and why outline drawings are so readily recognized as objects: The brain adds surfaces to outlines in the same way as it adds

(unperceived) colored surfaces to (perceived) changes in color and brightness across edges.

While the visual field is inherently grainy (an array of retinal receptor cells), the phenomenal world is smooth. That we are unaware of the graininess is another consequence of how the brain processes visual information. Not only are uniformly colored regions of the visual field filled in homogeneously on the basis of contrast information across boundaries [57], but also the graininess of the boundaries is glossed over. Cells that respond to specifically oriented line segments in a particular part of the visual field receive input from cells with lined-up circular receptive fields. While the information coming from cells of the latter type is discrete, the perception of a line segment occurring when a cell of the former type is stimulated is continuous.

The phenomenal world thus is intrinsically a world of sharp and continuous boundaries filled with homogeneous content. Unlike the physical world, it *is* constructed from boundaries, and for this reason it conforms to the CCP [58]. And if we naively believe that the physical world is constructed in the same way, we cannot but fail to make sense of quantum mechanics.

To see just how insidiously the CCP prevents us from decrypting quantum mechanics, let us examine some of its consequences. The idea that the synchronic multiplicity of the world depends on boundaries, or that the parts of a material object exist by virtue of the parts of the space it “occupies”, leads to the standard, substantive conception of an intrinsically and infinitely differentiated space. It is a substantive conception because it portrays space as intrinsically divided, and it does so because the parts of matter exist by virtue of the parts of space. (We cannot then hold the multiplicity of matter responsible for the multiplicity of space.)

Further, the parts of an intrinsically divided substantive space exist in themselves; each part—and in clear this means, each *conceivable* part—is a separate constituent of the world. But if *all* parts of space exist in themselves, there exist ultimate, not further divisible parts. (If each part were further divisible, we could not regard the division of space as completed. Hence we could not conceive of all parts of space as existing in themselves, independently of an always continuable process of division [59].) This is how the CCP tricks us into believing that space is adequately represented by (a point set cardinally equal to) \mathbb{R}^3 .

Here are some additional consequences of the CCP:

- Every material object has a form, defined as the boundary of the space it “occupies”.
- A material object has as many parts as the space it “occupies”. A noncomposite object therefore has the form of a point.
- Two material objects cannot “occupy” the same

space. Therefore all material objects are distinguishable by their positions.

- Numerically sharp distances exist between all points of space. The distances between two material objects is the distance between two points of space. Hence numerically sharp distances exist between all material objects.

As the present article has demonstrated, none of these consequences of the CCP is consistent with what quantum mechanics is trying to tell us. Yet, to my knowledge, no interpretation of quantum mechanics except the Pondicherry Interpretation [6] has ever challenged the CCP. This too goes to prove just how deep-seated a prejudice this fallacy is.

References and Notes

- [1] See for instance R.P. Feynman, R.B. Leighton, and M. Sands, *The Feynman Lectures in Physics*, Vol. 3 (Addison-Wesley, Reading, MA, 1965).
- [2] This classic thought experiment has been elegantly realized by A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki, and H. Ezawa, “Demonstration of single-electron buildup of an interference pattern”, *Am. J. Phys.* **57** (2), 117-120 (1989).
- [3] Reference [1], Sec. 1–1.
- [4] Richard P. Feynman, *The Character of Physical Law* (MIT Press, Cambridge, MA, 1967), p. 129.
- [5] David Z. Albert, *Quantum Mechanics and Experience* (Harvard University Press, Cambridge, MA, 1992), p. 11.
- [6] Ulrich Mohrhoff, “What quantum mechanics is trying to tell us”, *Am. J. Phys.* **68** (8), 728–745 (2000); Eprint quant-ph/9903051.
- [7] Ulrich Mohrhoff, “The One, the Many, and the Quantum”, Eprint quant-ph/0005110.
- [8] Michael Redhead, *Incompleteness, Nonlocality and Realism* (Clarendon, Oxford, 1987), p. 72.
- [9] “No elementary phenomenon is a phenomenon until it is a registered (observed) phenomenon.”—John Archibald Wheeler, “Law without law”, in *Quantum Theory and Measurement* (Princeton University Press, Princeton, NJ, 1983), edited by J.A. Wheeler and W.H. Zurek, pp. 182–213.
- [10] “No elementary quantum phenomenon is a phenomenon until it is registered, recorded, ‘brought to a close’ by an ‘irreversible act of amplification,’ such as the blackening of a grain of photographic emulsion or the triggering of a counter.”—John Archibald

Wheeler, “On recognizing ‘law without law’ (Oersted Medal Response at the joint APS-AAPT Meeting, New York, 25 January 1983)”, *Am. J. Phys.* **51** (5), 398–404 (1983). It deserves mention that the characterization of a measurement on a system S as involving an “irreversible act of amplification” is inadequate. Amplification achieves nothing but the entanglement of S with a system A possessing a large number of degrees of freedom: Instead of a probability measure over the possible properties of S , we then have a joint probability measure over the possible properties of both S and A , which are statistically correlated. No amount of amplification can take us from a probability assignment to a property-indicating fact. On the other hand, once a property-indicating event or state of affairs has happened or come into existence, it is *logically* impossible to reverse this. For the relevant fact is not that the needle deflects to the left (which can be reversed by returning the needle to its neutral position); the relevant fact is that *at the time t* the needle deflects (or points) to the left. This is a timeless truth. If at the time t the needle deflects to the left, then it always has been and always will be true that at the time t the needle deflects to the left.

- [11] Quantum mechanics presupposes a classical domain encompassing property-indicating events or states of affairs, and therefore it can neither be inconsistent with nor account for the existence of this domain. How to think consistently about the coexistence of the classical and quantum domains has been explained in Refs. [6] and [7].
- [12] The functional dependence of a probability measure $\psi(\mathbf{r}, t)$ on t is different from the time dependence of an actual state of affairs. A probability measure is not something that exists and evolves in time, any more than it is something that exists in space. $\psi(\mathbf{r}, t)$ is “prepared” by a complete measurement, and the time to which it refers is the time of a subsequent measurement.
- [13] I use “material object” synonymously with “physical object”.
- [14] There is one conceivable spatial property that is not expressible in terms of the spatial relations that obtain in the physical world, namely the allegedly pointlike form of a fundamental particle. As the following will show, this pointlike form is not among the spatial properties of the physical world.
- [15] Even in the phenomenal world, positions are *quantitatively* realized as relative positions. A single object can define “here” qualitatively, but it is insufficient for quantifying (attributing a numerical value to) “here”. For this purpose we need to know how “here” is quantitatively related to “elsewhere”.
- [16] Georg Cantor, *Gesammelte Abhandlungen* (Springer, Berlin, 1932), edited by A. Fraenkel and E. Zermelo, p. 204.
- [17] Frank Jackson, “What Mary didn’t know”, *J. Philos.* **83**, 291–295 (1986).
- [18] Immanuel Kant, *Critique of Pure Reason*, first (German) edition, 1781, p. 25.
- [19] E.g.: “These atomic laws can be only imprecisely transposed into visual images of the atom, for Planck’s quantum hypothesis on which these laws are based contains on principle a non-visualizable element.”—Werner Heisenberg, *Wandlungen in den Grundlagen der Naturwissenschaft* (Hirzel, Zürich, 1949), p. 47.
- [20] Barry Loewer, “Copenhagen versus Bohmian interpretations of quantum theory”, *Brit. J. Phil. Sci.* **49**, 317–328 (1998).
- [21] Letter to Høffding, 22.9.1923, Bohr Scientific Correspondence, microfilm No. 3, p. 5; in John Honner, “The transcendental philosophy of Niels Bohr”, *Stud. Hist. Phil. Sci.* **13** (1), 1–29 (1982).
- [22] Henry Pierce Stapp, “The Copenhagen interpretation”, *Am. J. Phys.* **40** (8), 1098–1116 (1972).
- [23] t is defined as the time at which the distinctions we make between the regions R_i are real for the electron. If these distinctions have no reality for the electron then neither has the time t . A *particular* time t is real for a quantum system only if it is the indicated time of possession of an indicated contingent property (such as being inside a particular region R_k), as is explained in Refs. [6] and [7].
- [24] As discussed in Ref. [6], the relevant facts may concern not only properties possessed before the time t but also properties possessed after this time. $p(R_i)$ is objective only if we make the additional assumption that there is only one relevant fact, indicating the possession of a specific energy value E_j prior to the time t .
- [25] Reference [8], p. 48.
- [26] “Difficult though it be to imagine physics either completable or incompletable, it is perhaps even more difficult to imagine that physics should be possible at all... Why can the multiplicity of events be subjected to the consequences of a few simple postulates?”—Carl Friedrich von Weizsäcker, *The Unity of Nature* (Farrar, Straus and Giroux, New York, 1980), pp. 174–175.

- [27] “The most incomprehensible thing about the world is that it is comprehensible.”—Einstein in *Albert Einstein: Philosopher-Scientist* (Library of Living Philosophers, Evanston, IL, 1949), edited by P.A. Schilpp, p. 112.
- [28] Albert Einstein, Boris Podolsky, and Nathan Rosen, “Can quantum-mechanical description of physical reality be considered complete?,” *Phys. Rev.* **47**, 777–780 (1935); reprinted in Wheeler and Zurek (Ref. [9]), pp. 138–141.
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- [30] Marlan O. Scully, Berthold-Georg Englert, and Herbert Walther, “Quantum optical tests of complementarity”, *Nature* **351** (6322), 111–116 (1991).
- [31] Berthold-Georg Englert, Marlan O. Scully, and Herbert Walther, “The duality in matter and light”, *Scientific American* **271** (6), 56–61 (December 1994).
- [32] Ulrich Mohrhoff, “Objectivity, retrocausation, and the experiment of Englert, Scully and Walther”, *Am. J. Phys.* **67** (4), 330–335 (1999).
- [33] If a process is capable of following several alternatives from one set $\{\mathcal{A}_k^1\}$ of mutually contradictory alternatives, there is at least one other set $\{\mathcal{A}_k^2\}$ of such alternatives that is complementary to $\{\mathcal{A}_k^1\}$. In an ESW experiment (Refs. [30]–[32]), for instance, the experimenters may determine either the slit taken by an atom (by identifying the microwave cavity containing the photon left behind by the atom) or the phase relation with which the atom emerged from the “undivided union” of the slits (by opening the electro-optic shutters and watching the photosensor situated between the cavities). The pair of slits and the pair of possible phase relations are complementary alternatives. If the experimenters determine neither the slit taken nor the phase relation, then not only the conceptual distinctions we make between the alternatives of each set correspond to nothing in the physical world, but also the conceptual distinction we make between the two sets of alternatives has no counterpart in the physical world. In this case we cannot add the amplitudes associated with the first set of alternatives (the atom takes this or that slit), for this would imply that the atom went through both slits with a definite phase relation, or that the conceptual distinction we make between the alternatives of the second set corresponds to something in the physical world.
- [34] Bertrand Russell, *A History of Western Philosophy* (Simon and Schuster, New York, NY, 1945), pp. 201–202.
- [35] It might be held that it is their spatial relations that tie particles together as members of a single ontology; without those relations, they would not exist in the same sense or in the same world. In truth, as we have seen in Sec. 7, the particles would not be *many* without those relations; they would be altogether what they intrinsically are—identically the same thing \mathcal{X} . In the absence of spatial relations there is nothing that could be tied together.
- [36] “[A]ll there is to the world is a vast mosaic of local matters of particular fact, just one little thing and then another. . . . We have geometry: a system of external relations of spatiotemporal distance between points. . . . And at those points we have local qualities: perfectly natural intrinsic properties which need nothing bigger than a point at which to be instantiated. . . . And that is all. . . . All else supervenes on that.”—David K. Lewis, *Philosophical Papers*, Vol. II (Oxford University Press, New York, NY, 1986), p. X.
- [37] Robert Audi, *The Cambridge Dictionary of Philosophy* (Cambridge University Press, Cambridge, 1995).
- [38] Carl Friedrich von Weizsäcker, “Parmenides and quantum theory”, in Ref. [26], pp. 379–400.
- [39] C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation* (W.H. Freeman and Company, San Francisco, 1973).
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- [41] David Bohm, “A suggested interpretation of quantum theory in terms of hidden variables”, *Phys. Rev.* **85**, 166–193 (1952); reprinted in Wheeler and Zurek (Ref. [9]), pp. 369–396.
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- [46] Hilary Putnam, *Representation and Reality* (MIT Press, Cambridge, MA, 1988), p. 113.
- [47] Bernard d’Espagnat, *Veiled Reality* (Addison-Wesley, Reading, MA, 1995).

- [48] Michael Lockwood, *Mind, Brain and the Quantum* (Basil Blackwell, Oxford, 1989).
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- [50] Fritz London and Edmond Bauer, “The theory of observation in quantum mechanics”, in Wheeler and Zurek (Ref. [9]), pp. 217–259.
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- [53] Henry Pierce Stapp, *Mind, Matter, and Quantum Mechanics* (Springer, Berlin, 1993).
- [54] N. David Mermin, “What is quantum mechanics trying to tell us?”, *Am. J. Phys.* **66** (9), 753–767 (1998).
- [55] D.H. Hubel and T.N. Wiesel, “Brain mechanisms of vision”, *Sci. Am.* **241** (3), 150–162 (1979).
- [56] Edwin H. Land, “The Retinex theory of color vision”, *Sci. Am.* **237** (6), 108–128 (1977).
- [57] Since this involves the transition from objective brain mechanisms to subjective visual percepts, just how the filling in is accomplished is presently as impenetrable as the question of how anything material can be conscious in the first place.
- [58] Recently José Luis Bermúdez (*The Paradox of Self-Consciousness*, MIT Press, Cambridge, MA, 1998) has marshaled a remarkable amount of evidence supporting the existence of a prelinguistic, non-conceptual, representational content of consciousness. On Bermúdez’s reading the evidence suggests that the prelinguistic infant perceives bounded segments of the world that are unified and coherent, that have certain properties (for instance, shape), and that stand in certain relations with each other. This too goes a long way towards establishing the innateness of the CCP.
- [59] This touches on the age-old conflict between the intellectual demand for completeness and the perception of continuity, which argues against it. Science being driven by the desire to know how things are in themselves, it tends to come down in favor of completeness. Quantum mechanics teaches us how these apparently conflicting demands are, in fact, consistent. What makes them seem inconsistent is again—what else could it be?—the CCP.