Glossary of Terms

Adapted from Moon, F. C. 1987, Chaotic Vibrations (New York: John Wiley & Sons).

Attractor: A set of points or a subspace in phase space toward which a time history approaches after transients die out. For example, equilibrium points or fixed points in maps, limit cycles, or a toroidal surface of section for quasiperiodic motions, are all classical attractors.

Cantor set: Formally, a set of points obtained on a unit interval by throwing out the middle third and iterating this operation on the remaining intervals. This operation, when carried to the limit leads to a fractal set of points on the line with dimension (ln2/ln3).

Chaotic: Denotes a type of motion that is sensitive to changes in initial conditions. A motion for which trajectories starting from slightly different conditions diverge exponentially with time. A motion with positive Lyapunov exponent.

Equilibrium point: In a continuous dynamical system, a point in phase space toward which a solution may approach as transients decay. In mechanical systems, this usually means a state of zero acceleration and velocity. For maps, equilibrium points may come in a finite set where the system visits each point in a sequential manner as the map or difference equation is iterated. Also called a fixed point.

Feigenbaum number: A property of a dynamical system related to the period-doubling sequence. The ratio of successive differences between period-doubling bifurcation parameters approaches the number 4.669…. This property and the Feigenbaum number have been discovered in many physical systems in the prechaotic regime.

Fractal: A geometric property of a set of points in an n-dimensional space having the quality of self-similarity at different length scales and having noninteger fractal dimension less than n.

Fractal dimension: The fractal dimension is a quantitative property of a set of points in an n-dimensional space which measures the extent to which the points fill a subspace as the number becomes very large.

Henon map: A set of two coupled difference equations with one quadratic nonlinearity. When one parameter is set equal to zero, the equations resemble the logistic or quadratic map.

Horseshoe map: A map of the plane onto the plane. Points in the lower half of a rectangular domain are stretched and contracted and mapped into a vertical strip in a section of the left-hand plane, while points in the upper half are stretched and contracted and mapped onto a strip in the right half-plane. The process is like transforming a rectangular domain into a horseshoe shaped set of points, hence the name. Similar to the baker's transformation. Repeated iterations can yield a fractal-like set of points.

Intermittency: A type of chaotic motion in which long time intervals of regular, periodic or stationary dynamical motion are followed by short bursts of randomlike motion. The time between bursts is not fixed but is unpredictable.

KAM theory: The initials stand for the theorists Kolmogorov, Arnold, and Moser who developed a theory regarding the existence of periodic or quasiperiodic motions in nonlinear Hamiltonian systems (i.e., systems that have no dissipation and in which the forces can be derived from a potential). The theory states that if small nonlinearities are added to a linear systems, the regular motions will continue to exist.

Limit cycle: In the engineering literature, a periodic motion that arises from a self-excited or autonomous system as in electrical oscillations. In the dynamical systems literature, it also includes forced periodic motions.

Lorenz equations: A set of three first-order autonomous differential equations that exhibit chaotic solutions. The equations were derived and studied by E. N. Lorenz as a model of atmospheric convection. This set of equations is one of the principal paradigms for chaotic dynamics.

Mandelbrot set: If z is a complex variable, the quadratic map $z \rightarrow z^2 + c$ has more than one attractor. Fixing the initial conditions, one can vary the complex parameter c to determine the basin of attraction as a function of c. The basin boundary is fractal, and the basin is known as the Mandelbrot set.

Map, mapping: A mathematical rule that takes a collection of points in some n-dimensional space and maps them into another set of points. When the rule is iterated, a map is similar to a set of difference equations.

Period doubling: Refers to a sequence of periodic vibrations in which the period doubles as some parameter in the system is varied. In the classic model, these frequency-halving bifurcations occur at smaller and smaller intervals of the parameter. Beyond a critical value of the parameter, chaotic vibrations occur. This scenario of chaos has been observed in many physical systems, but it is not the only road to chaos.

Phase Space: In mechanics, phase space is an abstract mathematical space whose coordinates are generalized coordinates and generalized momenta. In dynamical systems governed by a set of first-order evolution equations, the coordinates are the state variables or components of the state vector.

Poincare section (map): The sequence of points in phase space generated by the penetration of a continuous evolution trajectory through a generalized surface of plane in the space. For a periodically forced, second-order nonlinear oscillator, a Poincare map can be obtained by stroboscopically observing the position and velocity at a particular phase of the forcing function.

Quasiperiodic: A vibration motion consisting of two or more incommensurate frequencies. (Two frequencies are said to be incommensurate if their ratio can is not equal to the ratio of two integers.)

Rayleigh-Benard convection: Circulatory patterns of motion in a fluid produced by a thermal gradient and gravitational forces. The chaos model of Lorenz attempted to simulate some of the dynamics of thermal convection.

Renormalization: A mathematical theory in functional analysis (a branch of mathematics) in which properties of some mathematical set of equations at one scale can be related to those at another scale by a suitable change of variables. Developed by Nobel Prize winning physicist K. Wilson. Used in the theory of quadratic maps to derive the Feigenbaum number.

Self-similarity: A property of a set of points in which geometric structure on one length scale is similar to that on another length scale.

Strange attractor: Refers to the attracting set in phase space on which chaotic orbits move. An attractor that is not an equilibrium point nor a limit cycle, nor a quasiperiodic attractor. An attractor in phase space with fractal dimension.

Surface of section: See Poincare section.

Taylor-Couette flow: The flow of a fluid between two rotating, concentric cylinders.

Torus (invariant): The coupled motion of two undamped oscillators is imagined to take place on the surface of a torus, with the circular motion around the small radius representing the oscillatory vibration of one oscillator and motion around the large radius representing the other oscillator. If the motion is periodic, then a closed helical trajectory will wind around the torus. If the motion is quasiperiodic, then the orbit will come close to all points on the torus.

Universal property (universality): A property of a dynamical system that remains unchanged for a certain class of nonlinear problems. For example, the Feigenbaum number relating the sequence of bifurcation parameters in period doubling is the same for a certain class of nonlinear, noninvertible, one-dimensional maps.

Van der Pol equation: A second-order differential equation with linear restoring force and nonlinear damping which exhibits a limit cycle behavior. The classical mathematical paradigm for self-excited oscillations.