P026 For Weeks 3 and 4 Fall 2007

READINGS:

Boccio notes on chaos - pages 41-66
Boccio notes on fractals - pages 1-20
Stewart - Chapters 9-13,17

WEBSITE: http://chaos.swarthmore.edu/courses/SOC026/index.html

QUESTIONS FOR THOUGHT AND DISCUSSION AS YOU READ and IN CLASS:

STEWART, CHAPTER 9: SENSITIVE CHAOS

1. According to Stewart, what was the conclusion of Ruelle and Takens regarding the Landau or Hopf-Landau scenario for the onset of turbulence?

2. What positive claim did Ruelle and Takens make regarding the onset of turbulence.

3. One can think about several questions regarding the experiment on Couette-Taylor flow that was performed by Harry Swinney and Jerry Gollub. What was the physical arrangement of the experiment? What was the experimental procedure? What were the results, and how did those results differ from expectations?

4. In what respects did the experiments of Harry Swinney and Jerry Gollub support the claims of Ruelle and Takens.

5. An important test of the picture of Ruelle and Takens is to exhibit a strange attractor in the results of a computer experiment on some model of a dynamical system. Ruelle, Takens and Packard proposed a scheme for using data for this purpose. Suppose that your data were the output of a computer solving the Lorenz equations. How would you organize and plot the data in order to exhibit the Lorenz attractor.

6. Likewise, how do you organize and collect the data from real measurements of a dripping faucet in order to show that there is a strange attractor in the dynamics.

7. What seem to be the most important issues in this chapter for Stewart?

STEWART, CHAPTER 10: FIG-TREES AND FEIGENVALUES

1. On what particular problem (or model) was Feigenbaum working when he made his famous discovery?

2. "Scaling" is portrayed as an important idea in Feigenbaum's work. What is scaling, and how does it manifest itself in the problem on which he was working? What is the connection between "scaling" and "self-similarity?"

3. What is the process of "renormalization," and how is it related to scaling and self-similarity?

4. Likewise, "universality" is also portrayed as an important idea here. What does universality mean in this context?

5. Are the results of Feigenbaum's work to be regarded as a discovery, a creation, or as something else?

6. How does Albert Libchaber's experiment seem to have altered the perception of Feigenbaum's work among physicists? Why should the experiment have made such a difference? In a similar way, how did the work of Swinney and Gollub alter the perception of Ruelle's work?

7. Edward Lorenz had been working on thermal convection years earlier. Why didn't the work of Lorenz stimulate interest in Feigenbaum's work in the way that Libchaber's experiment seems to have done.

8. The experiment of Harry Swinney and Jerry Gollub (described in Chapter 9 of Stewart) was a study of the onset of turbulence in Couette-Taylor flow. Libchaber's experiment was a study of the onset of turbulence in thermal convection. In what respects are these experiments and their results comparable? In what respects do they differ? Does this comparison suggest that the mechanism for the onset of chaotic behavior, i.e., turbulence in this case, is universal?

STEWART, CHAPTER 11: THE TEXTURE OF REALITY

1. In what respects would Benoit Mandelbrot appear to be unrepresentative of someone doing “normal mathematics?”

2. What is a fractal? What are the defining properties of a fractal? How do we recognize a fractal? Is the word a noun or an adjective? (And, are these four different questions?)

3. What are examples of systems that exhibit fractal behavior or fractal structure? What insights about such systems are gained when their fractal character is recognized?

4. Does a common sponge have a fractal structure? How would you decide?

5. Is there a connection between fractals and properties of the logistic map? If so, what is that connection?

6. What is the relevance of the study of fractals to the study of dynamical systems?

STEWART, CHAPTER 12: RETURN TO HYPERION

1. The Newtonian two-body problem concerns the orbit of a star (Sun) and a single planet. What is the relevance and role of the Newtonian two-body problem in connection with the investigations of the tumbling of Hyperion described by Stewart? In connection with investigations of the orbits of asteroids in the Kirkwood gaps? In connection with investigations of the stability of the solar system?

2. In the model of the tumbling of Hyperion that underlies Figure 106 on page 234, friction is neglected. Nevertheless, the role of friction is an important consideration in deciding that this is the appropriate model to investigate. What is the role of friction in this case, and how does it constrain the choice of the model to be investigated?

3. Is there a strange attractor in the chaotic region of the surface of section presented in Figure 106 on page 234? If there were a strange attractor, how would it be identified? Why might we conclude that there is no strange attractor here?

4. The chaotic tumbling of Hyperion does not involve a strange attractor. Why, nevertheless, would we conclude that the tumbling is chaotic?

STEWART, CHAPTER 13: THE IMBALANCE OF NATURE

1. The Oster model of population cycles includes cycles of periods three and six. What might we expect in view of that information?

Recall the discussion of climate and weather by Lorenz. The state of the system is described in terms of the values of quantities (e.g., temperature, pressure, wind speed and direction, relative humidity, etc.) that characterize the weather worldwide. The picture is that the dynamics of the global weather system might be described in terms of an attractor – presumably a strange attractor. If the atmosphere is approaching the attractor, then, although the weather is unpredictable in the long term, the attractor constrains the extremes of weather that might occur. If the attractor is stable, then the long-term climate (defined by the attractor) is stable and predictable, notwithstanding that the weather is unpredictable.

2. How does the picture described above for weather and climate translate into a characterization of population dynamics and an ecosystem as suggested in the section "The Web of Life" in Chapter 13? What are the quantities that would characterize the state of an ecosystem? What is the significance of the attractor? In such terms, how might one characterize the transformation of a tropical rain forest into a desert?

3. In the investigation of epidemics, how might one use the data illustrated in Figure 211 of Stewart in order to construct attractors like those illustrated in Figure 212? And how might one construct a Poincare map from the result?

4. As described by Stewart, is the kicked rotator a continuous dynamical system described by a differential equation or a system of differential equations, or is it a discrete dynamical system described by a difference equation or system of difference equations (i.e., an iterated map)?

STEWART, CHAPTER 17, FAREWELL, DEEP THOUGHT

This final chapter contains reflections on the subject of order and chaos. The following questions are a complement to Stewart's reflections. We shall make use of the chapter and questions as bases for the general discussion with which we shall conclude the course.

1. How might we define or describe chaos? What are some of the manifestations of chaotic behavior in dynamical systems?

2. How has the study of chaos altered the landscape of science? Have the laws of physics and chemistry changed? The principles of ecology? What is it that has changed?

3. What is the role of mathematics in the study of the natural sciences? Of computation? Have these roles been altered by the study of chaotic behavior?

4. How do we learn things about the natural world?

5. Are causal connections or patterns more important in the study of chaotic behavior?

6. Is chaotic behavior a single, universal phenomenon? Is there a universal route to chaos in a physical system? Just what is universal about chaotic behavior?