P002A 1st-Year Seminar Assignment #1 Fall 2007

READINGS:	**RESPONSIBILITY**
Boccio notes on chaos - pages 1-18	**Boccio**
Stewart - Chapters 1-4 (63 pages)	________&________
Ruelle - Chapters 3-5 (20 pages)	________&________

WEBSITE: http://chaos.swarthmore.edu/courses/SOC002a/index.html

LECTURE : Boccio - Boccio notes on chaos - pages 1-18

QUESTIONS FOR THOUGHT AND DISCUSSION:

DETERMINISTIC SYSTEMS, RANDOM PROCESSES, AND CHAOS ACCORDING TO IAN STEWART AND DAVID RUELLE ________&________

A. General Considerations

1. What do the two authors accomplish in the introductory chapters of their books?
2. What should we understand as the meanings of the terms "deterministic systems," "random processes," and "chaos?" Do Stewart and Ruelle agree on the meanings of these terms?
3. What might be examples of a deterministic system, a random process, and chaos?
4. The book Chaos by James Gleick has the subtitle Making a New Science. Is the study of order and chaos a science? Or is the study of order and chaos a part of a particular scientific discipline such as astronomy, biology, mathematics, physics, or another discipline?

B. Deterministic Systems in the Natural World

1. What is the function of mathematics in the study of the natural world?
2. What does the discovery of "laws of nature" contribute to an understanding of the natural world?
3. In what ways are mathematical formulations of the laws of nature useful? In particular, for what purposes do we reduce the laws of nature to differential equations? (By the way, what is a differential equation? As a student of liberal learning, try to answer this question without making use of technical aspects of mathematics.)
4. What is the connection between the reduction of the laws of nature to differential equations and the conclusion that those laws describe deterministic systems?

C. The Encounter of Voyager 1 and 2 with Hyperion.

1. Give a precise description of the hypothetical experiment presented in the section “Voyage to Hyperion” in Does God Play Dice?
2. What is the expected result of that experiment? Why?
3. What is the “observed” result?
4. What is the apparent paradox revealed by a comparison of the expected and observed results?
5. What does this discussion suggest about the relationship between the determinism of a physical system and the predictability of that system?

D. Introducing Chaos

1. Stewart introduces the reader to chaotic behavior in the first chapters of his books. How does he do it?
2. He illustrate his introductions to chaos by describing the behaviors of particular dynamical systems or models of dynamical systems. What are those illustrative examples?
3. Is this a good system with which to illustrate chaotic behavior? Why?
4. In what respects are systems governed by discrete mappings and systems governed by differential equations similar? In what respects are they different?
5. Ruelle reaches the point of introducing the subject of chaos by still a third, lengthier route. Manifestations of chaos first appear in Chapter 7, and an explicit discussion of chaos does not appear until Chapter 11. Why is it useful, nevertheless, to begin reading Ruelle now?

STEWART, CHAPTER 2: EQUATIONS FOR EVERYTHING ________&_________

A. Models of the Solar System

1. Geocentric models.
   a. Ptolemy.
   b. Antikythera mechanism.
2. Heliocentric models.
   a. Copernicus
   b. Kepler’s laws.
3. Tychonic model.
4. What does Stewart leave out of this account of models of the solar system?
5. What is the basis on which one would prefer one model to the others?

B. Dynamics

1. Galileo.
    a. Falling bodies.
    b. The pendulum.
    c. Galilean satellites of Jupiter; Kepler's third law.
2. Newton.
    a. Laws of motion.
    b. Law of gravitation.
    c. Calculus.
        (i) Differentiation.
        (ii) Integration.

C. Analytical Dynamics

1. Vibrations.
    a. Strings.
    b. Bells.
    c. Drums.
    d. Organ pipes.
2. Fluid dynamics.
3. Flow of heat.
4. How would mathematicians have studied such systems and determined their behaviors?
5. What is the paradigm for doing classical physics?

D. Other Issues

1. Often, solutions cannot be found exactly and in closed form.
2. Technical problems.
    a. Three-body collisions.
    b. Singularities.
3. Lagrangian and Hamiltonian formulations of mechanics.
4. Statistical problems.

E. Questions.

1. The chapter is essentially a history of astronomy and physics. What are the highlights?
2. What are the attributes of classical physics that are portrayed in the chapter?
3. What limits our power to study physical systems in this way?
4. What is the role of mathematics in all of this?
5. Why is the Copernican model of the solar system preferable to the Ptolemaic model?
6. What are the issues involved in the predicting the motion of a single planet around a star? (The two-body problem.)

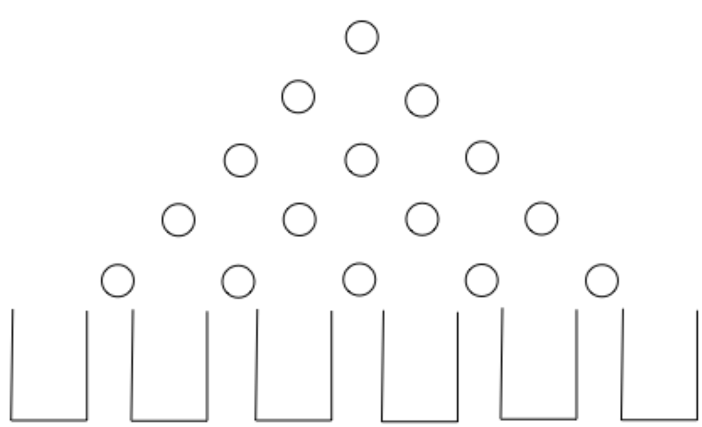
F. Elements of Calculus

1. Differentiation.
    a. Plot a curve representing a function.

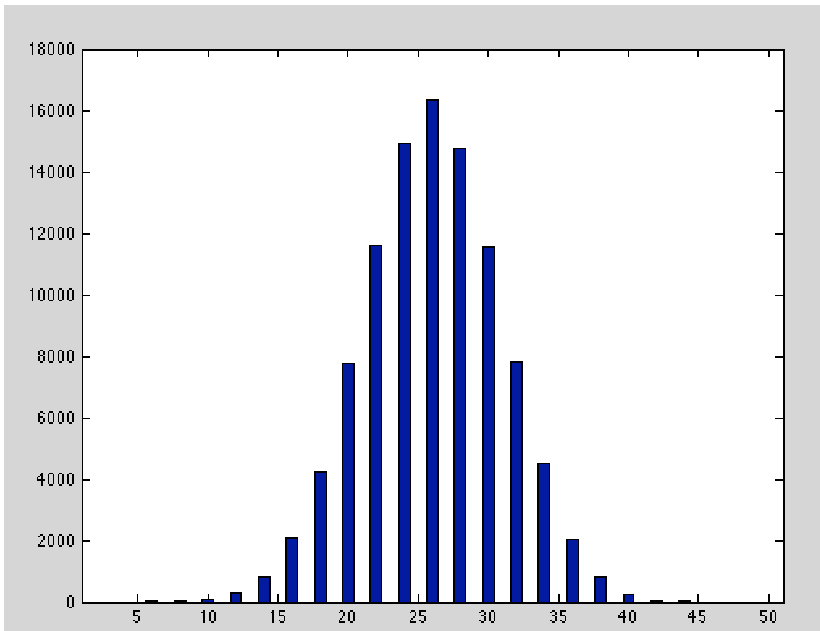
   b. Plot a straight line tangent to the curve at a point.
   c. Define the slope of the line.
   d. The derivative at the tangent point is the slope of the line.
2. Integration
   a. Plot velocity against time.
   b. Approximate the velocity curve in terms of line segments.
   c. Estimate the displacement of the particle that accumulates in a given interval of time as the sum of the areas under the line segments.
   d. Claim that we can improve the estimate by taking smaller intervals of time and a larger number of (smaller) line segments.
   e. In the limit, the accumulated displacement is the area under the velocity curve.
3. The point is not to do the mathematics. The point is to understand the claim that is based on doing the mathematics.

STEWART, CHAPTER 3: EQUATIONS FOR EVERYTHING AND RUELLE, CHAPTERS 3 THROUGH 5 ________&_________

In science museums, e.g. The Museum of Science and Industry, one often encounters a mechanical illustration of statistics with the aid of the following apparatus. Pegs are mounted in rows on a vertical board. The top row contains one peg, the second row two pegs, the third row three pegs, and so on, so that the pegs are arranged in a triangular array. A vertical channel leads to the top peg from above, and a row of bins lies below the base of the triangle that is formed by the lowest pegs. For the benefit of the viewer, the front wall of the apparatus is a sheet of Plexiglas. A sketch of the apparatus is shown below.



The elementary experiment to be performed with this device consists of dropping a ball into the channel leading to the top peg and observing the bin in which the ball arrives after it makes its way downward through the array of pegs. In a museum demonstration, many balls are dropped successively into the upper channel, and the viewer is asked to consider the relative numbers of balls that arrive in the different bins. The results of this demonstration are shown below



A. Analysis of the Apparatus as a Deterministic System

1. Consider the elementary experiment in which we introduce a single ball into the apparatus. Is this a deterministic dynamical system? Explain.
2. If you have concluded that it is not a deterministic system, then explain where the randomness arises.
3. If you have concluded that it is a deterministic system, then is it chaotic in a technical sense?

B. Analysis of the Apparatus in Terms of Probably and Statistics

1. What aspect of the behavior of the system might we describe in probabilistic terms? What values would you assign to the relevant probabilities?
2. What aspects of the museum demonstration with this apparatus would one describe in statistical terms.

STEWART, CHAPTER 4: THE LAST UNIVERSALIST ________&_________

A. Equilibrium and Stability

1. Consider a bowl of the form of a hemisphere and a ball bearing free to roll about in the bowel. Describe a situation in which the bearing is in static equilibrium (i.e., at rest). Is this a stable state of equilibrium? How can you tell?
2. Now turn the bowl upside down. Again the ball bearing is free to roll about on the inverted bowl. Is there a static equilibrium state in which the bearing is at rest? Describe it. Is this a stable state of equilibrium? How can you tell?
3. Now turn the bowl upright. If there were no friction or air resistance, then we could get the bearing to roll at a constant rate around the bowl on a horizontal circular trajectory. This would be a state of dynamical equilibrium in which the force of gravity, the centrifugal force, and the force exerted by the bowl just balance to keep the bearing in a steady motion. How would one test this state for stability? If the system were stable, then what would happen? What would happen if it were not stable?

B. Stability of the Solar System

1. Imagine that we could investigate the stability of the solar system experimentally with the aid of a time machine. In order to perform the experiment, we visit the solar system two billion years in the future. Describe the arrangement and motions of the planets that you would expect to observe if the solar system were stable.
2. Describe the arrangement and motions of the planets that you might expect to observe if the solar system were unstable.
3. Is this concept of the stability of the solar system the same as the concept considered above of the stability of states of the ball bearing free to roll on the surface of the bowl?

C. Poincare's Methods

1. In page 59, Stewart explains that Poincare represents the state of a dynamical system in terms of a point in "some huge-dimensional phase space." Moreover, the motion of the system is represented by a curve traced out by that point in the phase space. Consider a single planet moving in the gravitational field of the sun. Describe the phase space in which Poincare would

represent the motion of the planet. How many dimensions would that phase space have? What are those dimensions?

2. What are the principles or laws that determine the curve in the phase space representing the motion of the system considered in the preceding section?
3. Stewart then describes the use of a "Poincare section" in order to find periodic orbits of the system. For the case considered above of a single planet, describe a possible Poincare section. Suppose we could watch the point representing the state of the system move about in the phase space. How might we use the Poincare section in order decide whether or not the motion is a periodic orbit.
4. Stewart describes "Hill's reduced model" in terms of "Neptune, Pluto, and a grain of interstellar dust." What seems to be missing in this picture?
5. What are the "footprints of chaos" that Poincare found when he investigated a surface of section for Hill's reduced problem?